Reconstruction and Geometric Algorithms

Romain Vergne
Prashant Goswami
2013 - 2014
PLAN

• Introduction
• Parameterized / Ordered data
  – Curve Interpolation
  – Surface Interpolation
  – Curve Approximation
  – Surface Approximation
PLAN

• Introduction

• Parameterized / Ordered data
  – Curve Interpolation
  – Surface Interpolation
Point Interpolation
Parametric Functions

Linear
Shepard
Shepard
Polynomial
Point Interpolation
Parametric Functions

Hermite Interpolation
Point Interpolation
Parametric Functions

Curves
Spline
Point Interpolation

Parametric Functions

- Bilinear
- Shepard
- Polynomial
Point Interpolation
Parametric Functions

Tensor Product
Point Interpolation

Parametric Functions

Interpolation Spline Radial
Curve Interpolation
Parametric Functions

Linear

Hermite
Curve Interpolation
Parametric Functions

Coon Patch
Laplace Interpolation

Functions $f(x, y) = z$

$$\Delta f(x, y) = 0$$

Dirichlet constraints on boundaries

$$f(x_i, y_i) = z_i$$

Coon Patch
Laplace Interpolation

Functions $f( x, y ) = z$
Poisson Interpolation

Functions $f(x, y) = z$

\[
\Delta f(x, y) = \beta = \nabla f(x, y)
\]

Dirichlet constraints

\[
f(x_i, y_i) = z_i
\]

Neumann constraints

\[
\nabla f(x, y) = (g_x, g_y)^T
\]
PLAN

• Introduction
• Parameterized / Ordered data
  – Curve Interpolation
  – Surface Interpolation
  – Curve Approximation
  – Surface Approximation
Subdivision Curves

- Repeatedly refine the control polygon
  \[ P^1 \rightarrow P^2 \rightarrow P^3 \rightarrow P^4 \ldots \]
- Curve is the limit of an infinite process
Subdivision Curves

Chaikin subdivision: \(a = \frac{3}{4}, \ b = \frac{1}{4}\)
Subdivision Curves

Chaikin subdivision : $a = \frac{3}{4}$, $b = \frac{1}{4}$
Subdivision Curves

Chaikin subdivision: \( a = \frac{3}{4}, \ b = \frac{1}{4} \)
Subdivision Curves

Chaikin subdivision: $a = \frac{3}{4}, \ b = \frac{1}{4}$
Subdivision Curves

Chaikin subdivision : \( a = \frac{3}{4}, b = \frac{1}{4} \)
Subdivision Curves

Chaikin subdivision: $a = \frac{3}{4}$, $b = \frac{1}{4}$
Subdivision Curves

Chaikin subdivision: $a = 0.9$, $b = 0.25$
Subdivision Curves

Chaikin subdivision: $a = 0.9$, $b = 0.25$
Subdivision Curves

Chaikin subdivision: $a = 0.9$, $b = 0.25$
Subdivision Curves

Chaikin subdivision: $a = 0.9$, $b = 0.25$
Subdivision Curves

Chaikin subdivision: $a = 0.9$, $b = 0.25$
Subdivision Curves

Chaikin subdivision : $a = 0.9$, $b = 0.25$
Subdivision Curves

Catmull-Clark subdivision
Subdivision Curves

Catmull-Clark subdivision
Subdivision Curves

Catmull-Clark subdivision
Subdivision Curves

Catmull-Clark subdivision
Subdivision Curves

Catmull-Clark subdivision
Subdivision Curves

Catmull-Clark subdivision
Subdivision Curves

Catmull-Clark subdivision interpolation
Subdivision Curves

Catmull-Clark subdivision interpolation
Subdivision Curves

Catmull-Clark subdivision interpolation
Subdivision Curves

Catmull-Clark subdivision interpolation
Subdivision Curves

Catmull-Clark subdivision interpolation
Subdivision Curves

Catmull-Clark subdivision interpolation
Subdivision Curves

Chaikin 1

Chaikin 2

Catmull-Clark (approx)

Catmull-Clark (interp)
Subdivision Surfaces

Triangle Surface

Subdivision Loop
Subdivision Surfaces

Catmull-Clark Subdivision
Subdivision Surfaces

Triangulated Surface

Butterfly Subdivision
Subdivision Surfaces

Triangulated Surface

Modified Butterfly Subdivision
Videos

• Loop subdivision
• Catmull-Clark
• Butterfly subdivision
## Comparison

<table>
<thead>
<tr>
<th>Nom</th>
<th>Maillage</th>
<th>Type</th>
<th>Continuité</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loop</td>
<td>Triangle</td>
<td>Approx</td>
<td>$C^2$ ($C^1$ s.e.)</td>
</tr>
<tr>
<td>$\sqrt{3}$</td>
<td>Triangle</td>
<td>Approx</td>
<td>$C^2$ ($C^1$ s.e.)</td>
</tr>
<tr>
<td>Catmull-Clark</td>
<td>Quadrangle</td>
<td>Approx</td>
<td>$C^2$ ($C^1$ s.e.)</td>
</tr>
<tr>
<td>Doo-Sabin</td>
<td>Quadrangle</td>
<td>Approx</td>
<td>$C^1$ ($C^0$ s.e.)</td>
</tr>
<tr>
<td>Butterfly</td>
<td>Triangle</td>
<td>Interp</td>
<td>$C^1$ ($C^0$ s.e.)</td>
</tr>
<tr>
<td>Butterfly modif.</td>
<td>Triangle</td>
<td>Interp</td>
<td>$C^1$</td>
</tr>
<tr>
<td>Kobbelt</td>
<td>Quadrangle</td>
<td>Interp</td>
<td>$C^1$</td>
</tr>
</tbody>
</table>
Least Squares

___ : $y = ax + b$
Least Squares

Equation: \( y = ax + b \)
Least Squares

\[ y = ax + b \]
Least Squares

Gaussian

\[ y = ax + b \]
Weighted Least Squares

Parabola: $y = ax^2 + bx + c$
Weighted Least Squares

Parabola: $y = ax^2 + bx + c$
Weighted Least Squares

Cubic: $y = ax^3 + bx^2 + cx + d$
Cubic: \[ y = ax^3 + bx^2 + cx + d \]
Bibliography

• Cours de Nicolas Szafran 2011/2012
  (http://www-ljk.imag.fr/membres/Nicolas.Szafran/)

• Scattered Data Interpolation and Approximation for Computer Graphics
  (Siggraph Asia course 2010)

• Implicit surface reconstruction from point clouds (Johan Huysmans’s thesis)