Reconstruction and Geometric Algorithms

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Prashant Goswami

2013 - 2014

PLAN

- Introduction
- Parameterized / Ordered data
  - Curve Interpolation
  - Surface Interpolation
  - Curve Approximation
  - Surface Approximation

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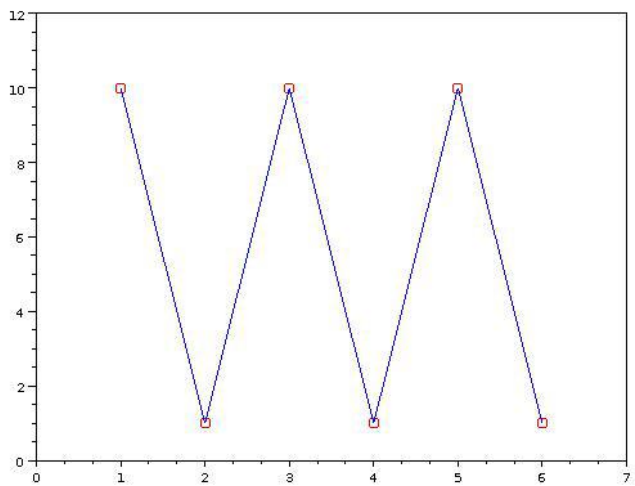
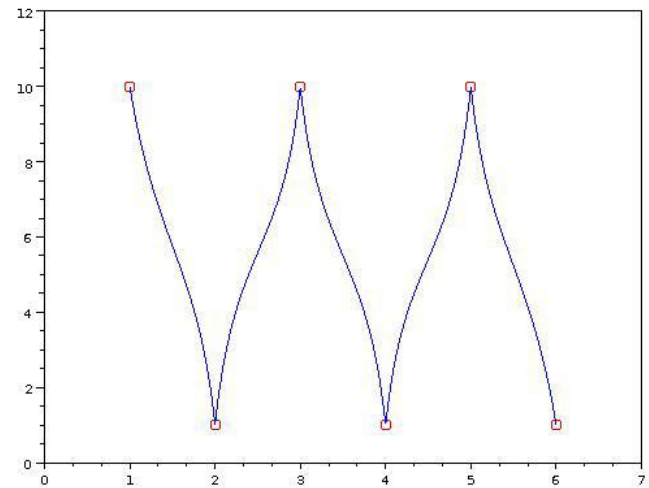
Point Interpolation

Parametric Functions



Linear



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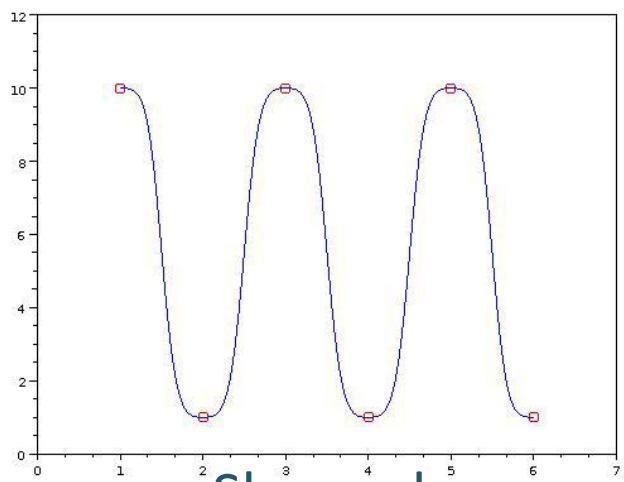

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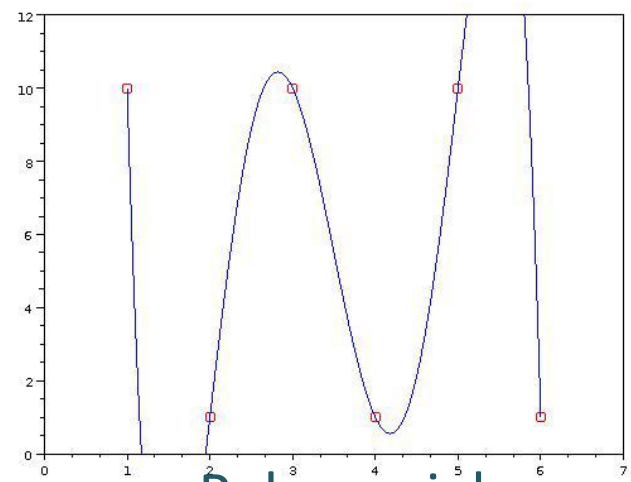
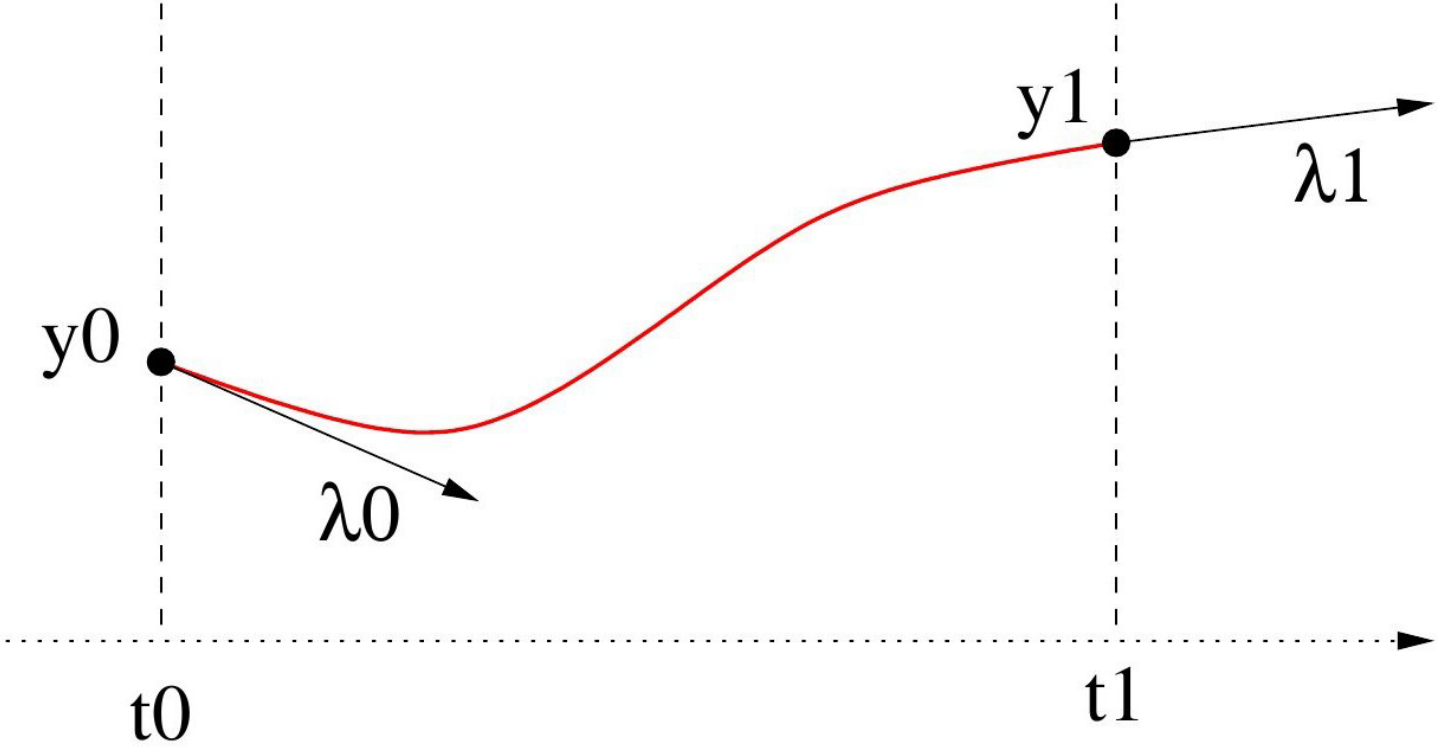


Polynomial

Point Interpolation

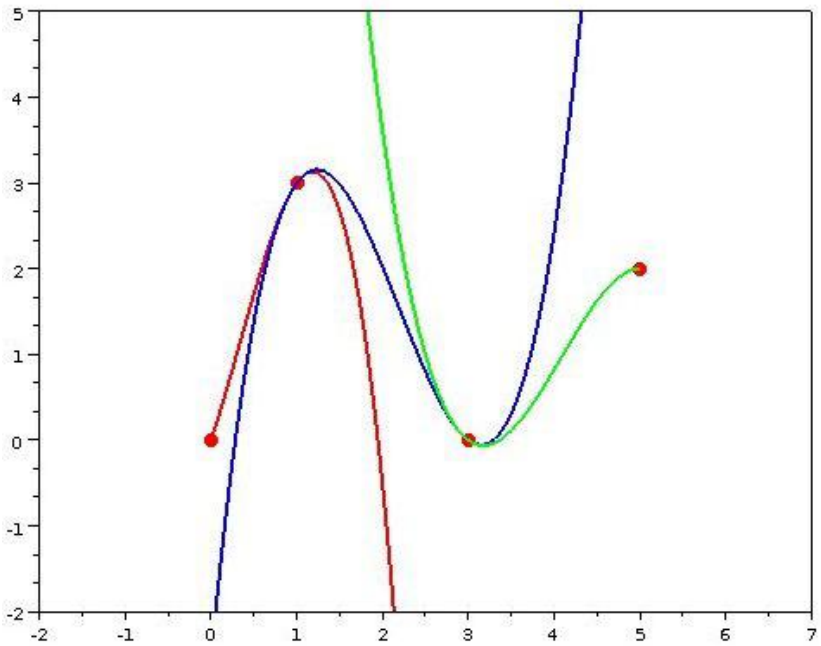
Parametric Functions



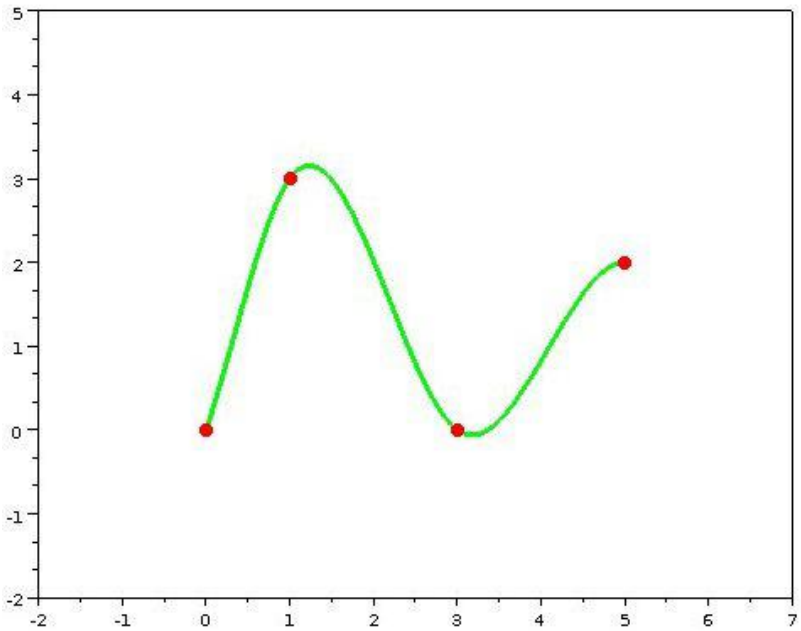
Hermite Interpolation

Point Interpolation

Parametric Functions



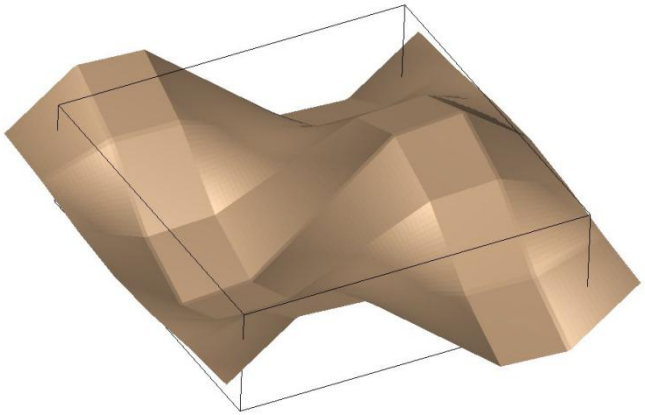
Curves



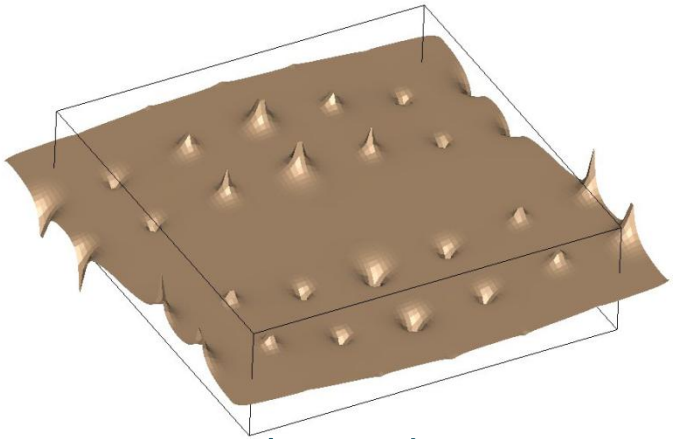
Spline

Point Interpolation

Parametric Functions



Bilinear



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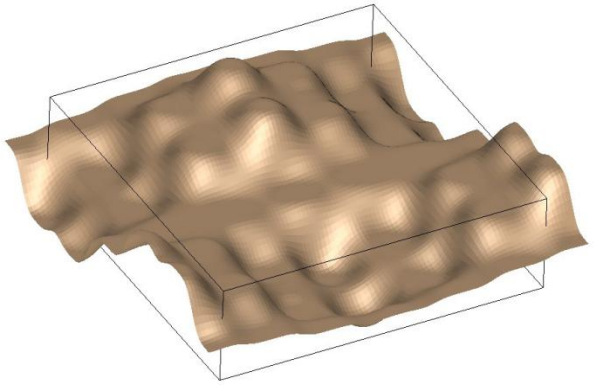

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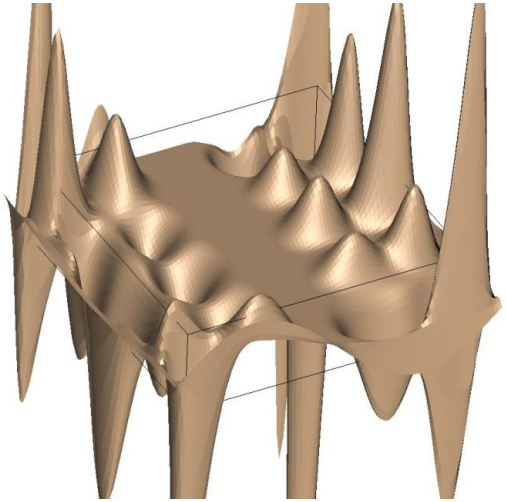
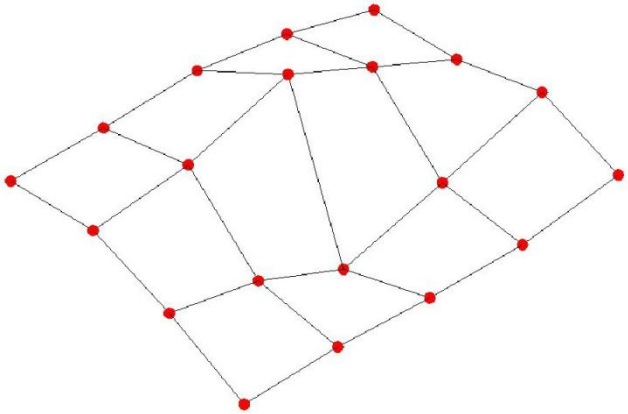
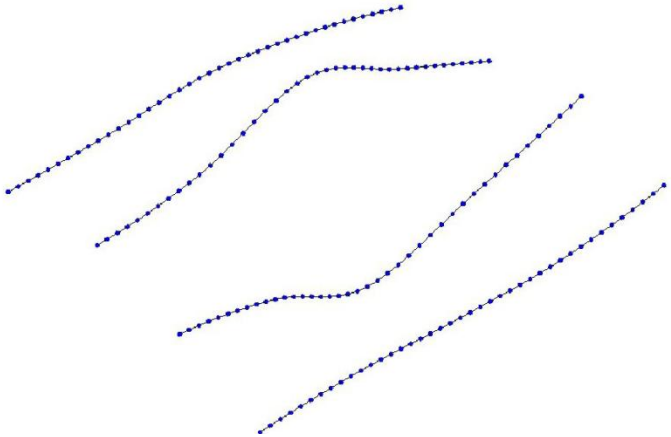


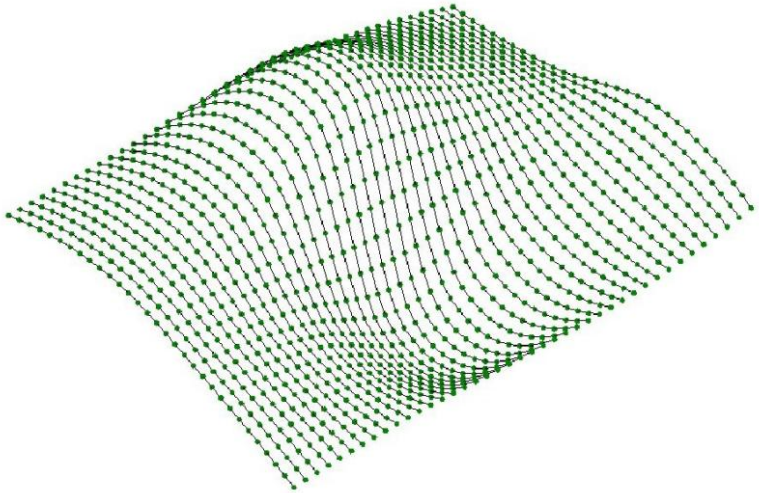
Polynomial

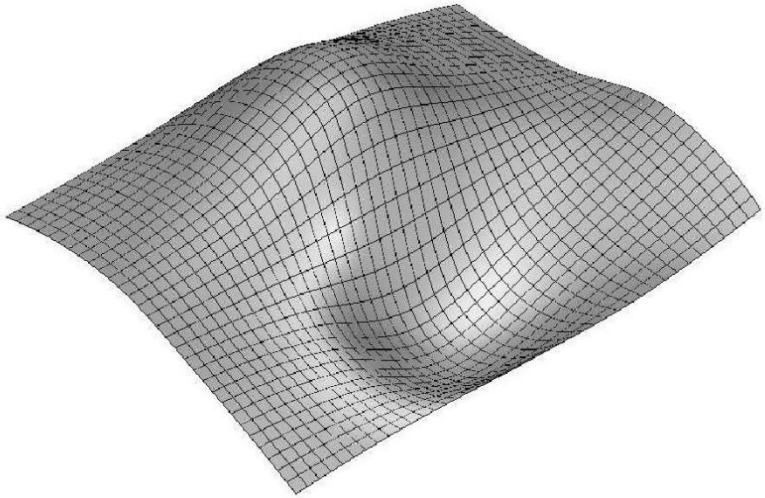
Point Interpolation

Parametric Functions





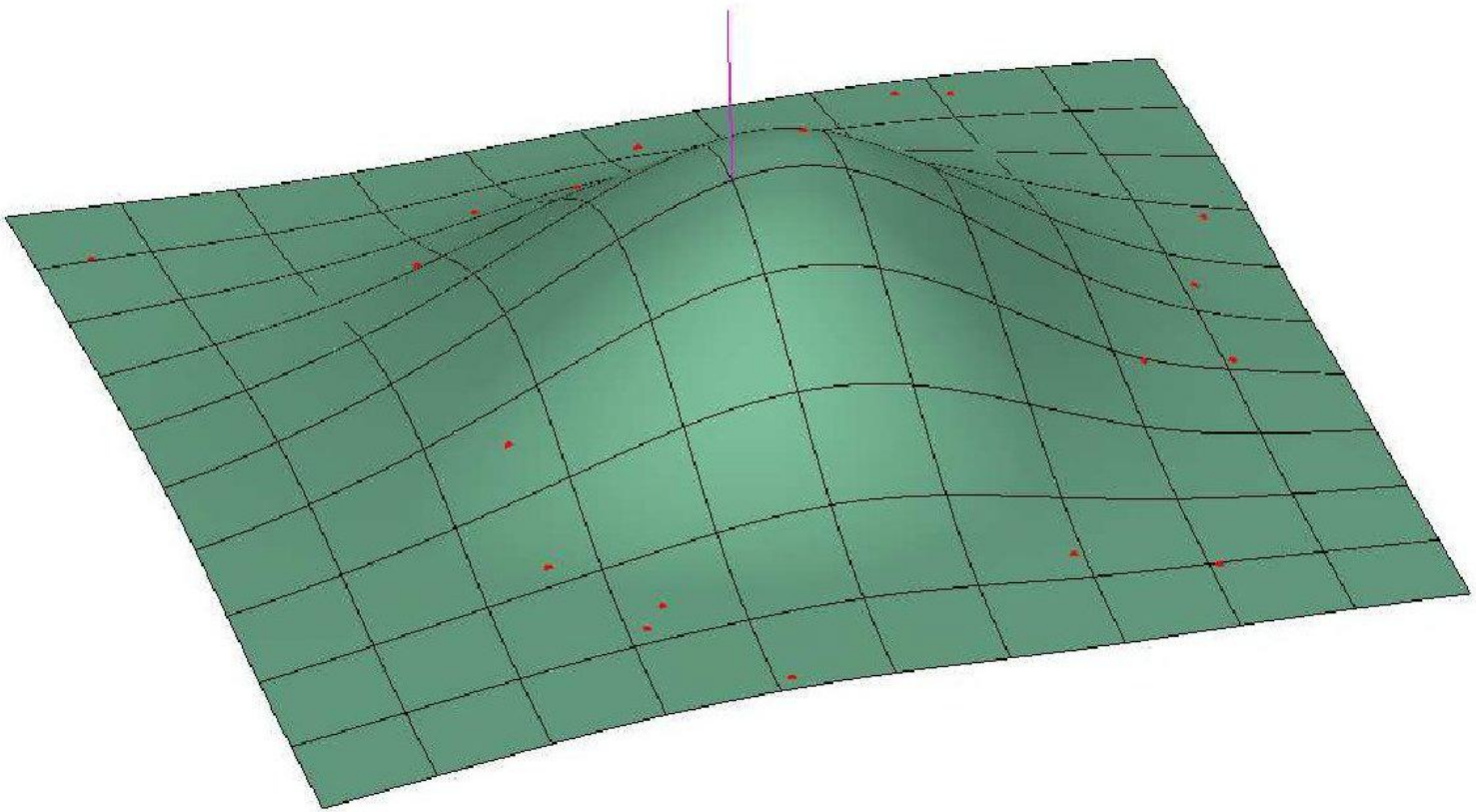




Tensor Product

Point Interpolation

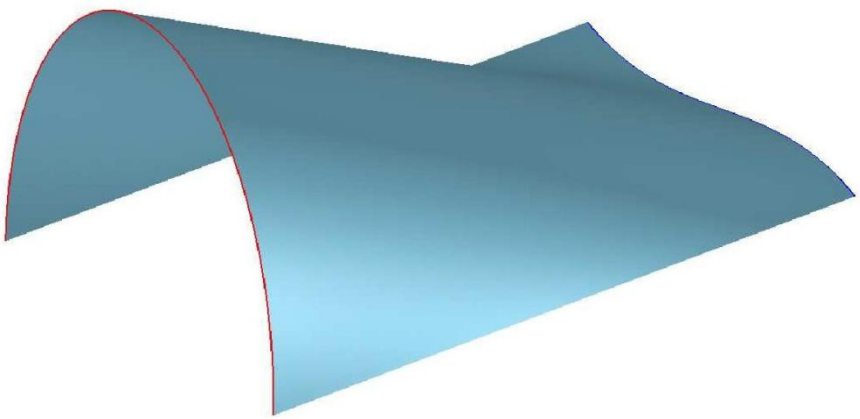
Parametric Functions



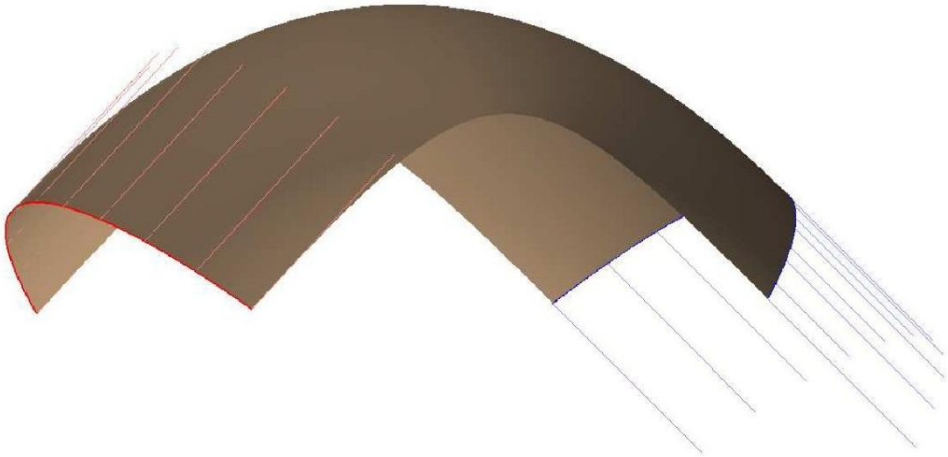
Interpolation Spline Radial

Curve Interpolation

Parametric Functions



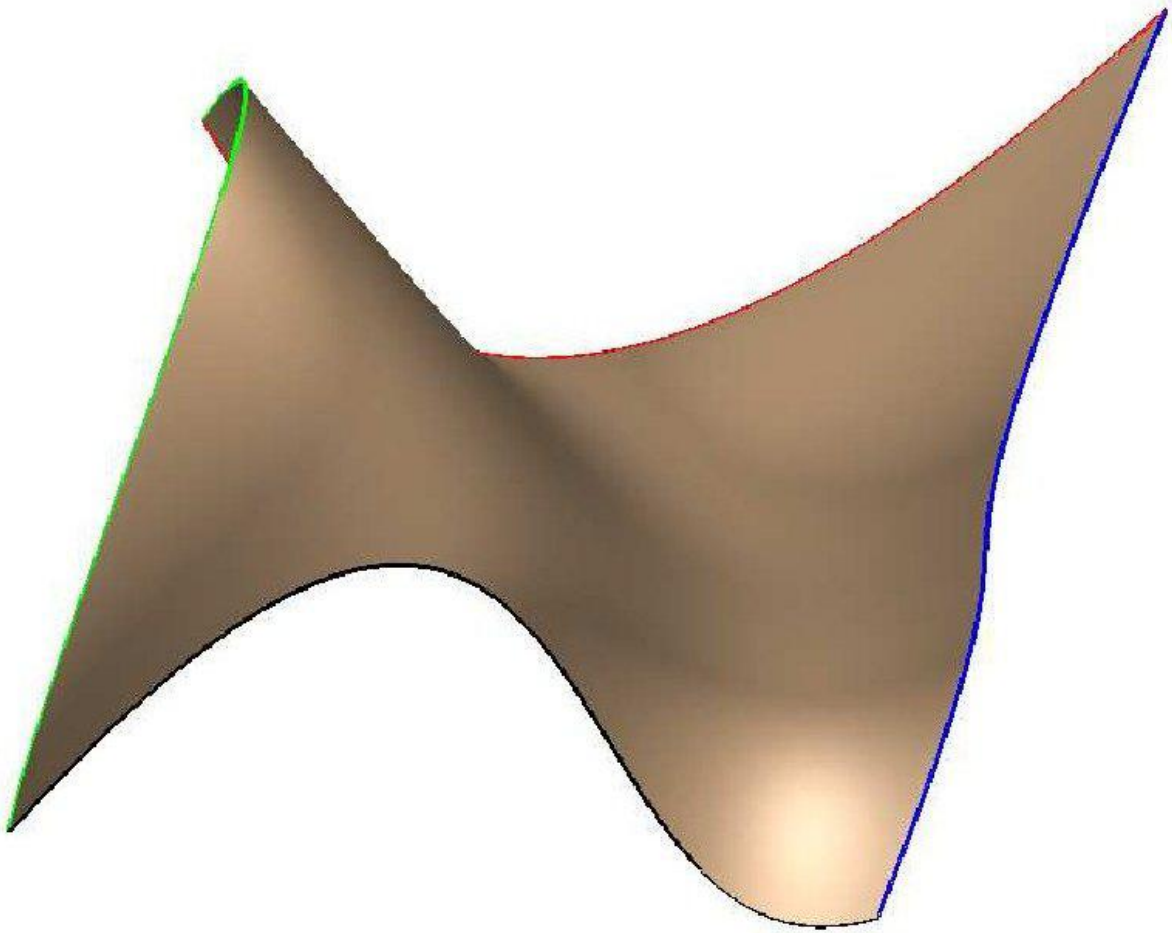
Linear



Hermite

Curve Interpolation

Parametric Functions



Coon Patch

Laplace Interpolation

Functions *f(x, y) = z*

$$\Delta f(x, y) = 0$$

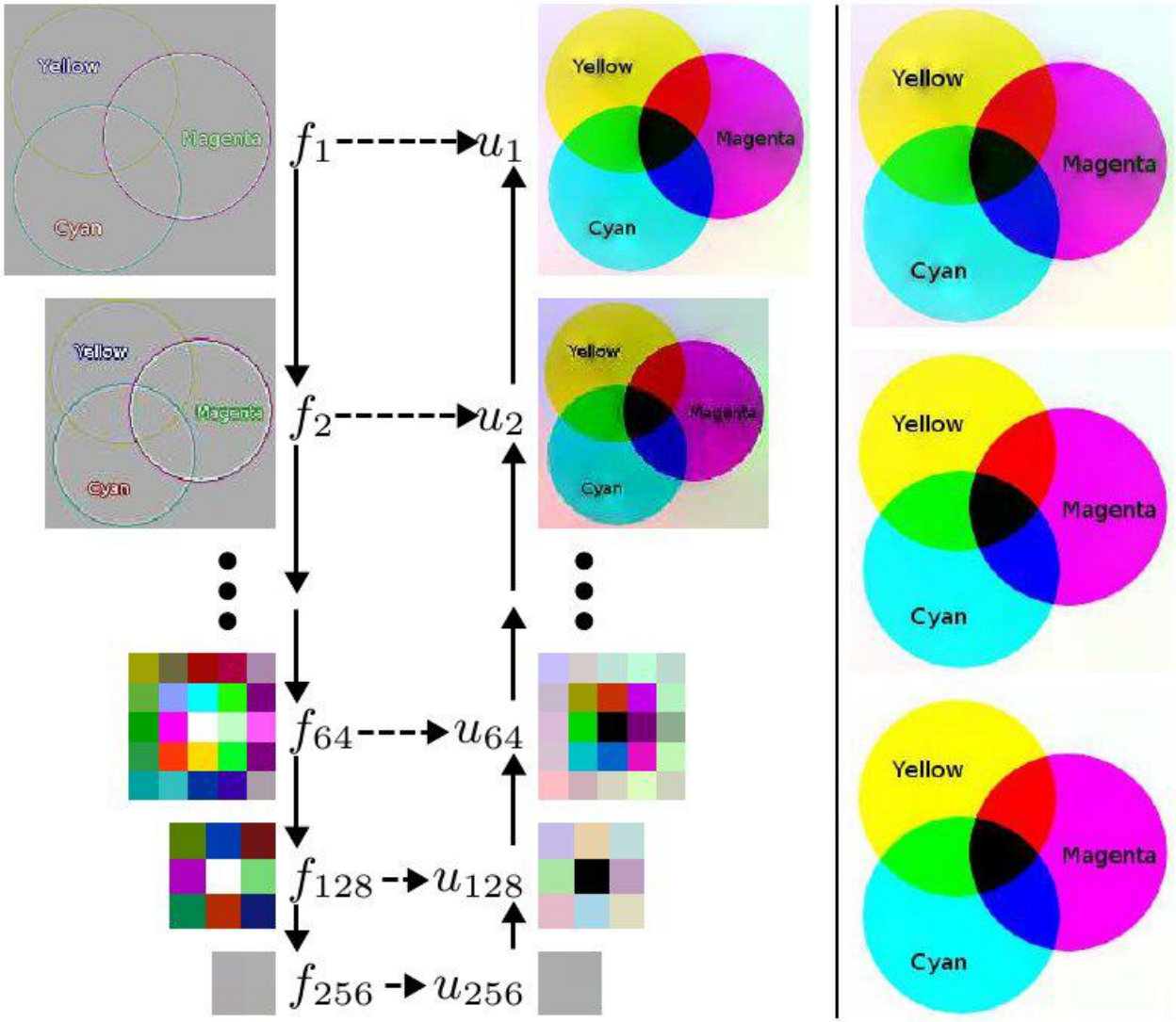
Dirichlet constraints on boundaries

$$f(x_i, y_i) = z_i$$

Coon Patch

Laplace Interpolation

Functions *f(x, y) = z*



Poisson Interpolation

Functions *f(x, y) = z*

$$\Delta f(x, y) = \beta = \nabla f(x, y)$$

Dirichlet constraints

$$f(x_i, y_i) = z_i$$

Neumann constraints

$$\nabla f(x, y) = (g_x, g_y)^T$$

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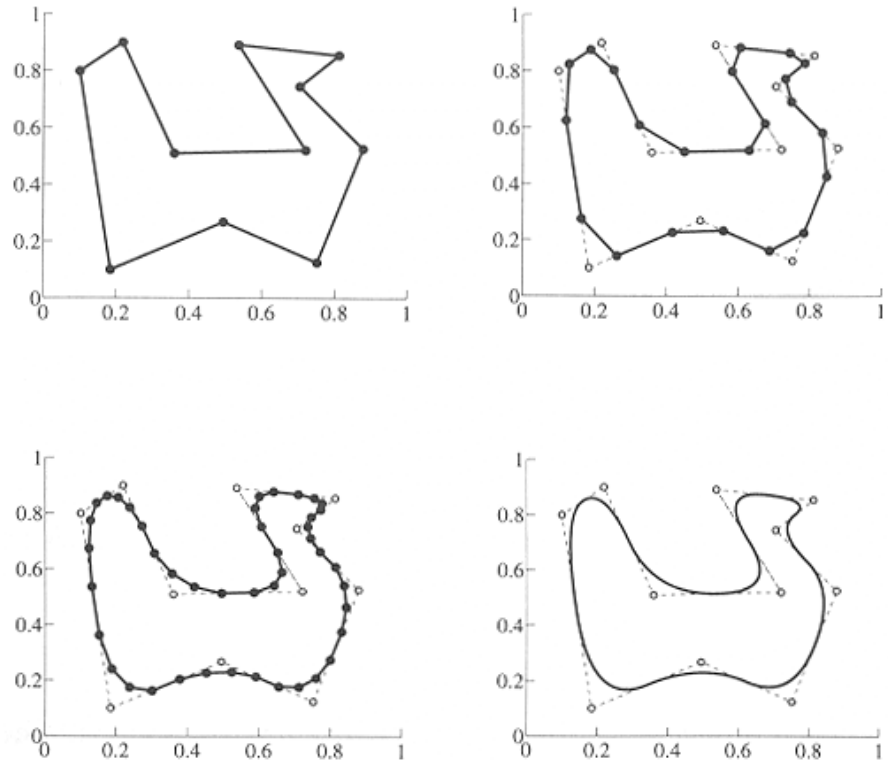
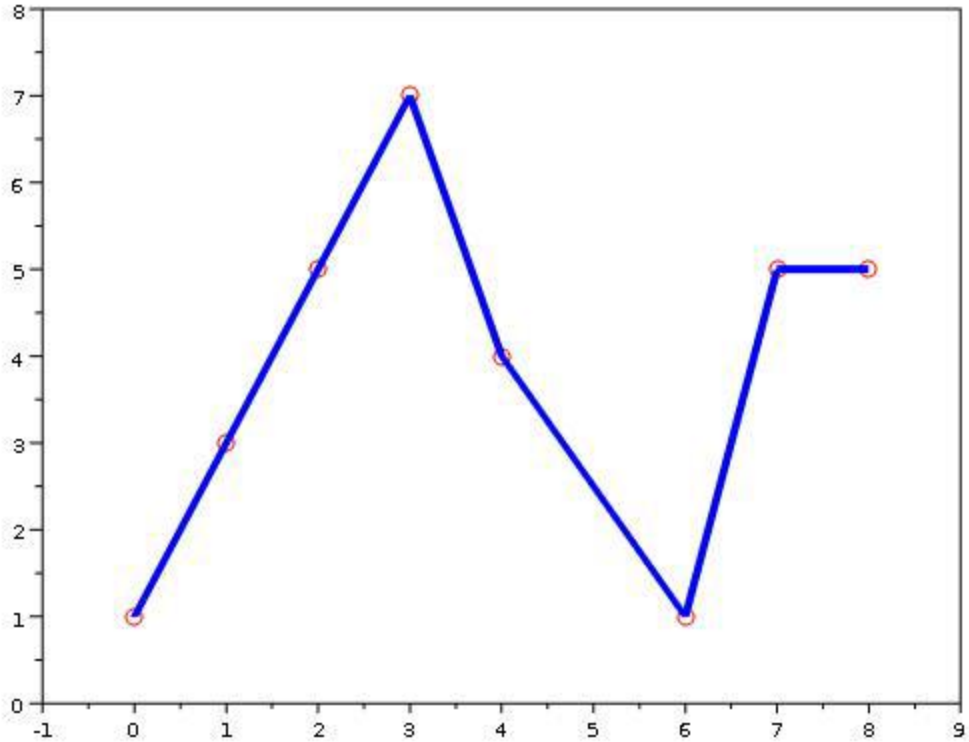
Subdivision Curves

- Repeatedly refine the control polygon
  - $P^1 \rightarrow P^2 \rightarrow P^3 \rightarrow P^4$ …….
- Curve is the limit of an infinite process

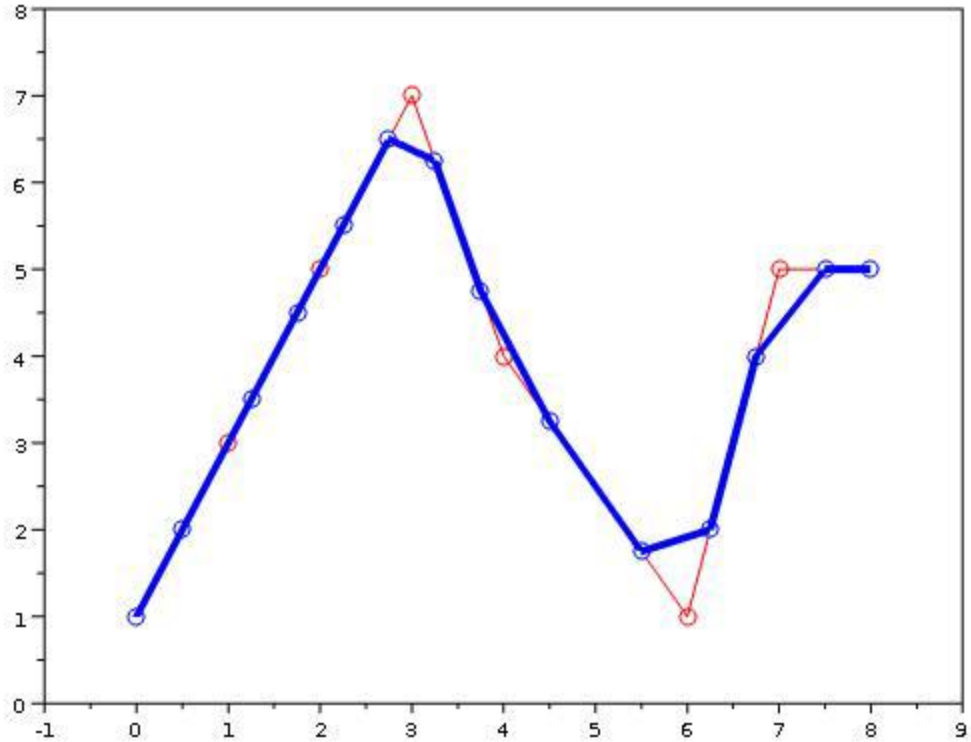


Subdivision Curves



Chaikin subdivision : a = ¾ , b = ¼

Subdivision Curves



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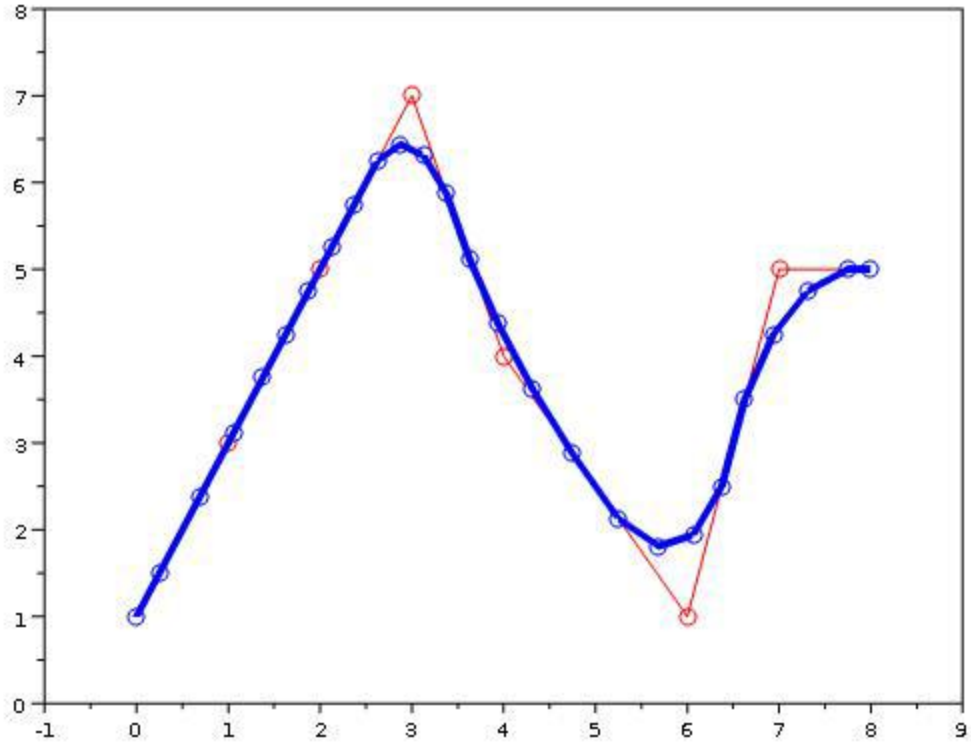

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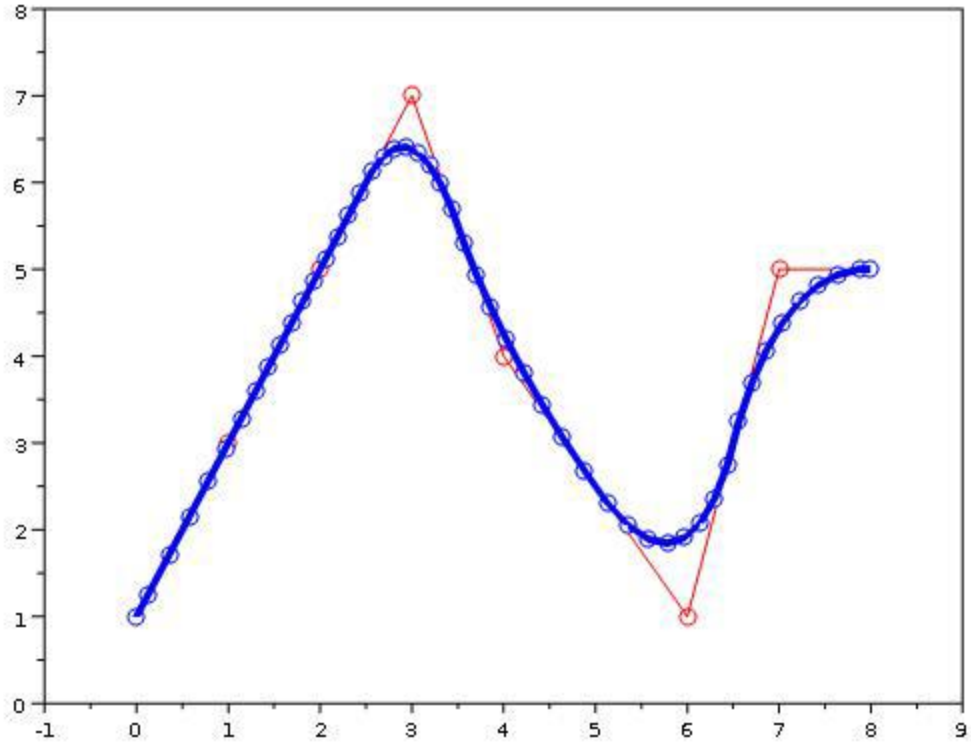

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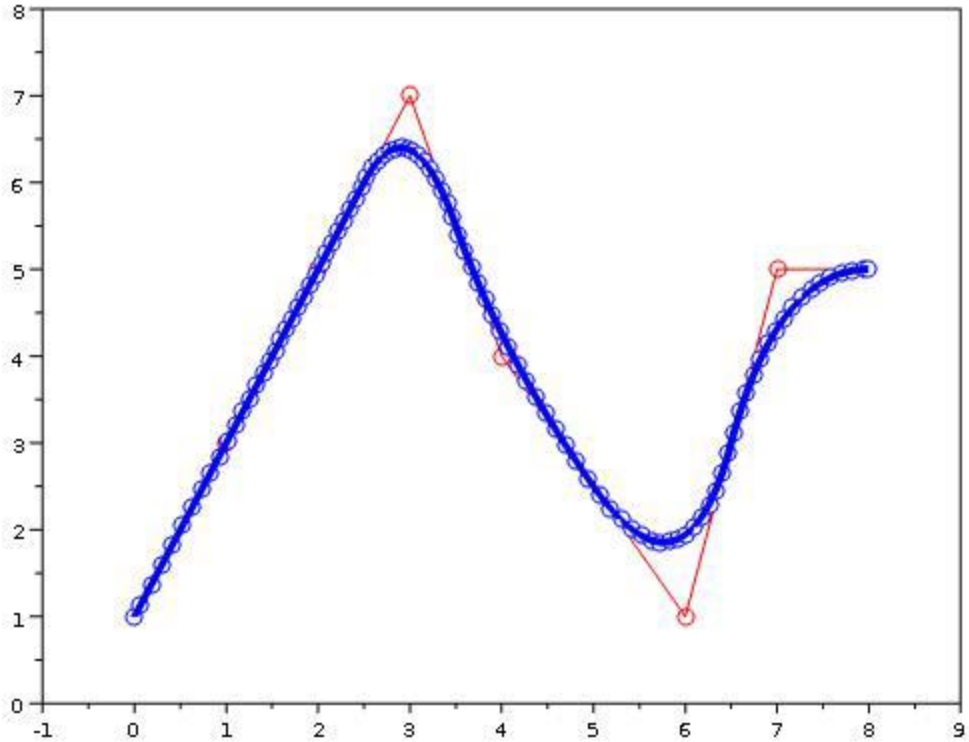

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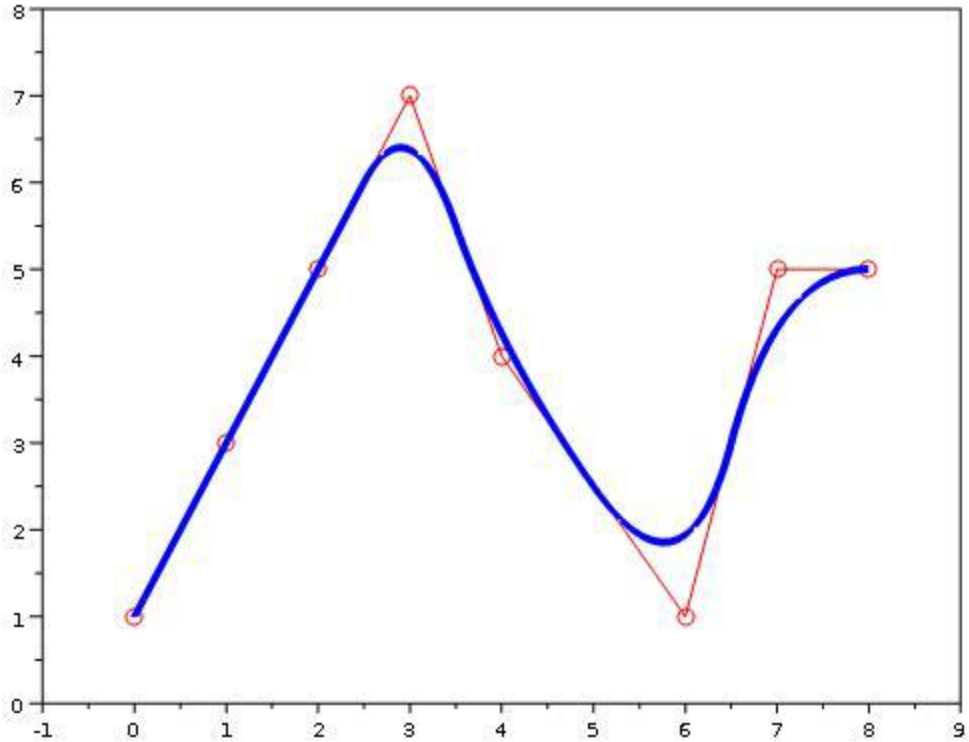

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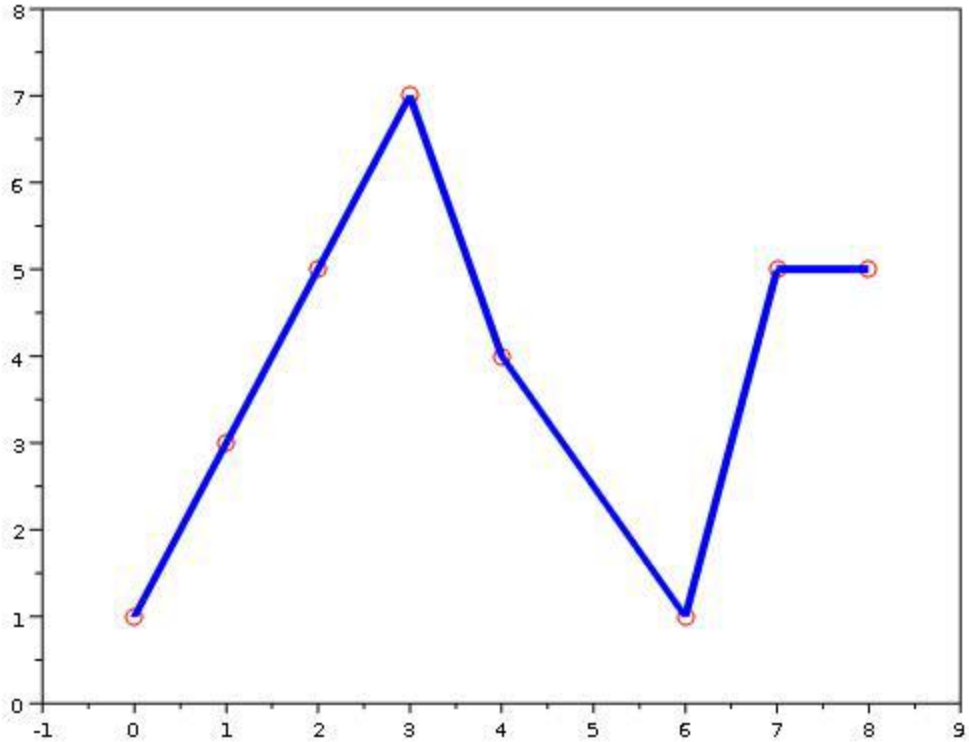
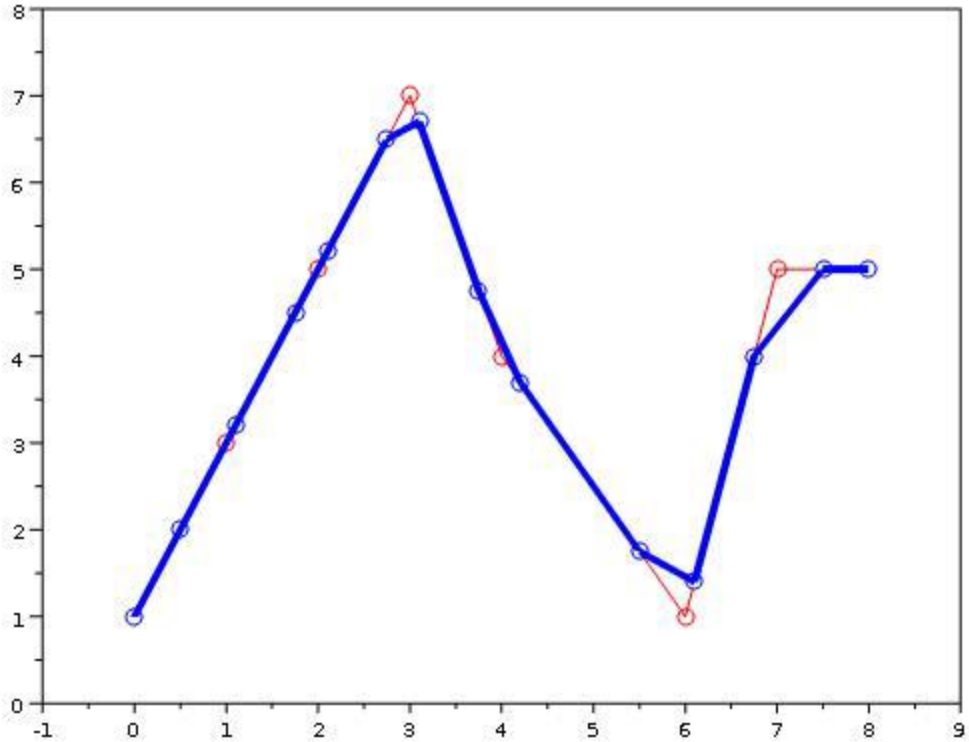


Chaikin subdivision : a = 0.9 , b = 0.25

Subdivision Curves



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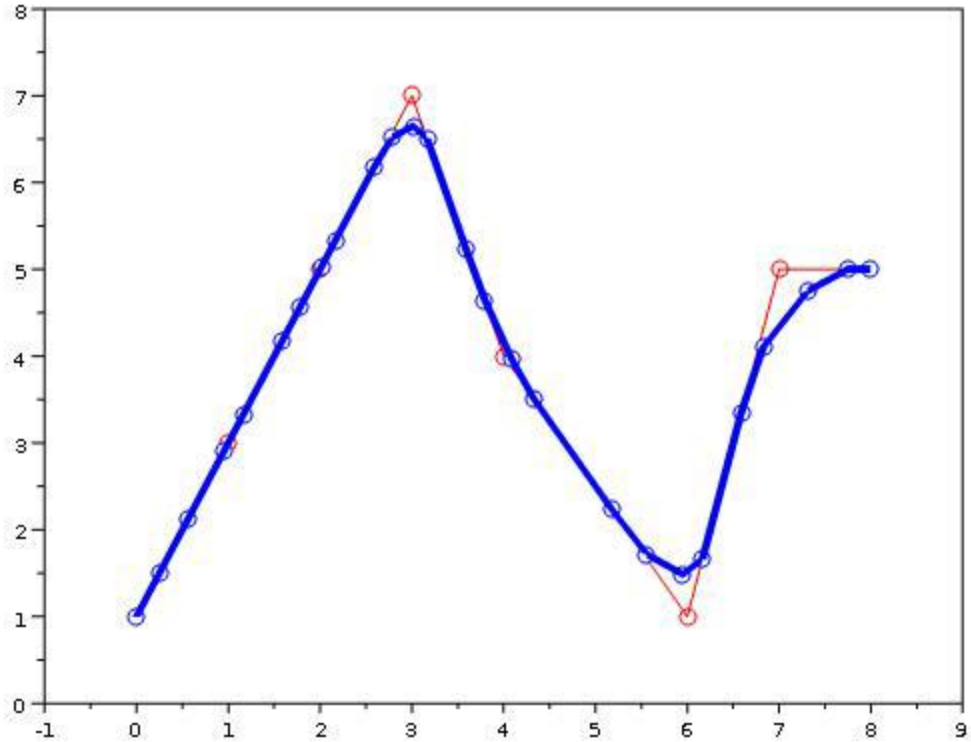

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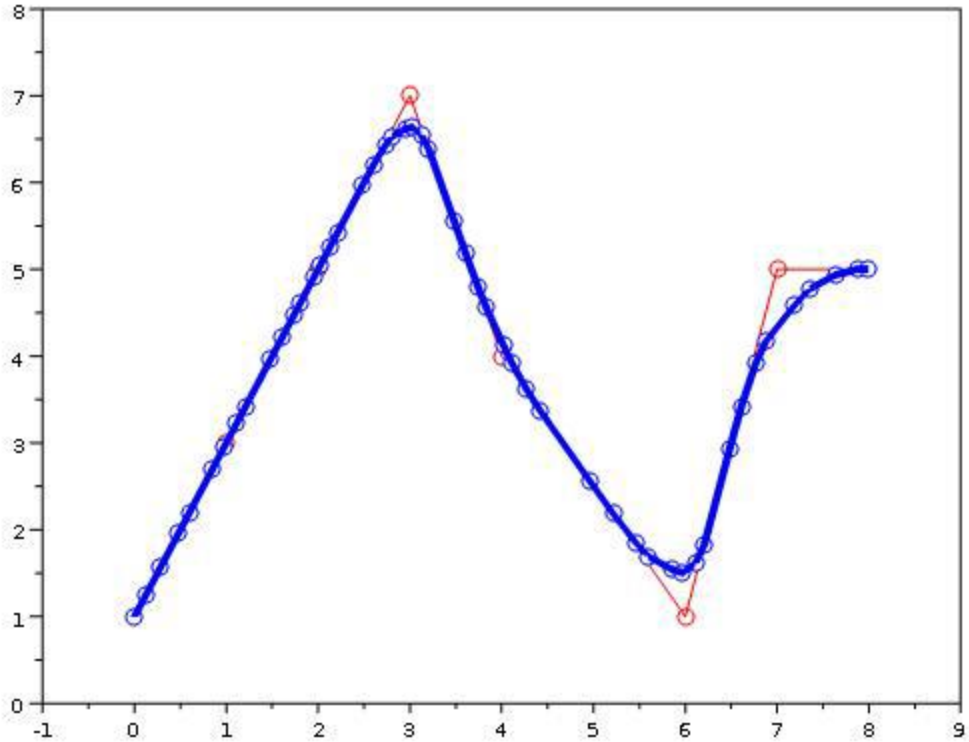

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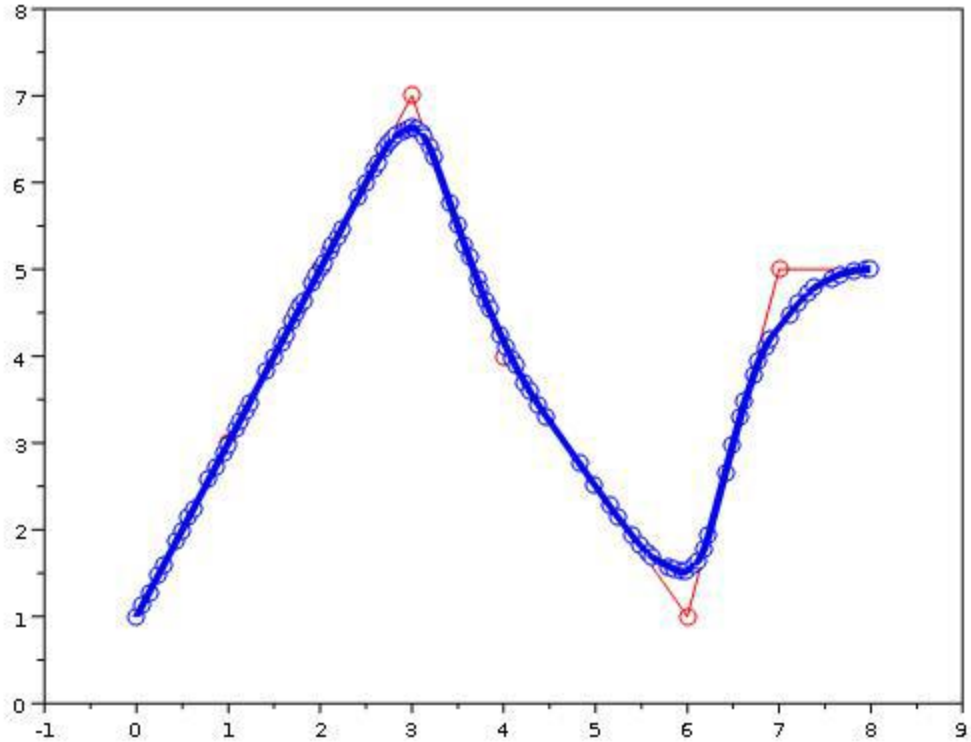

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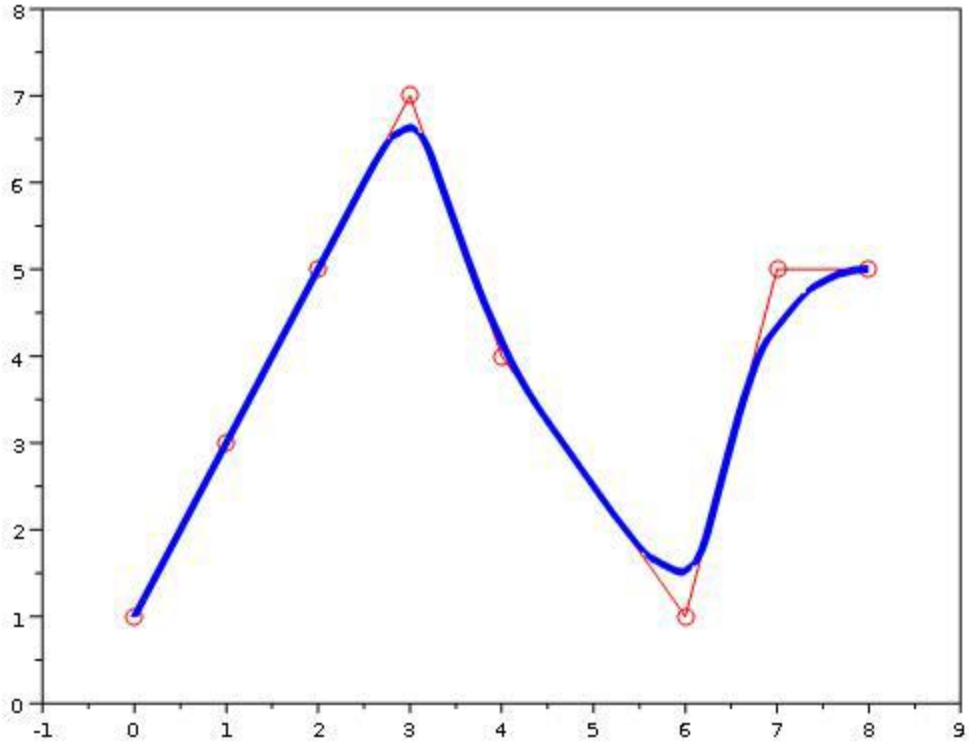

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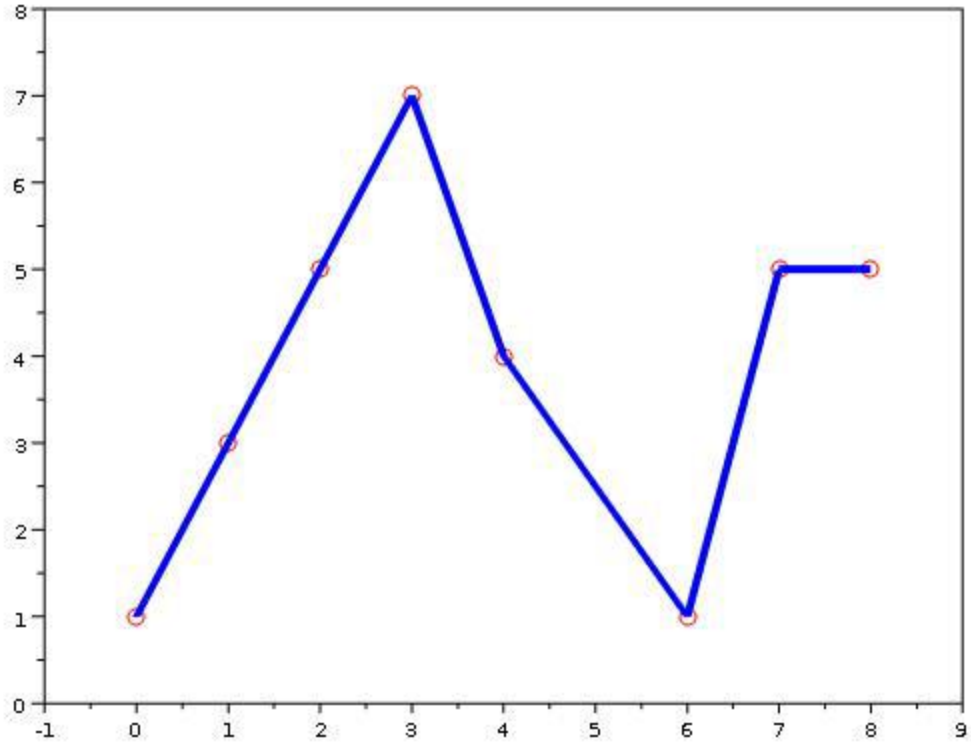
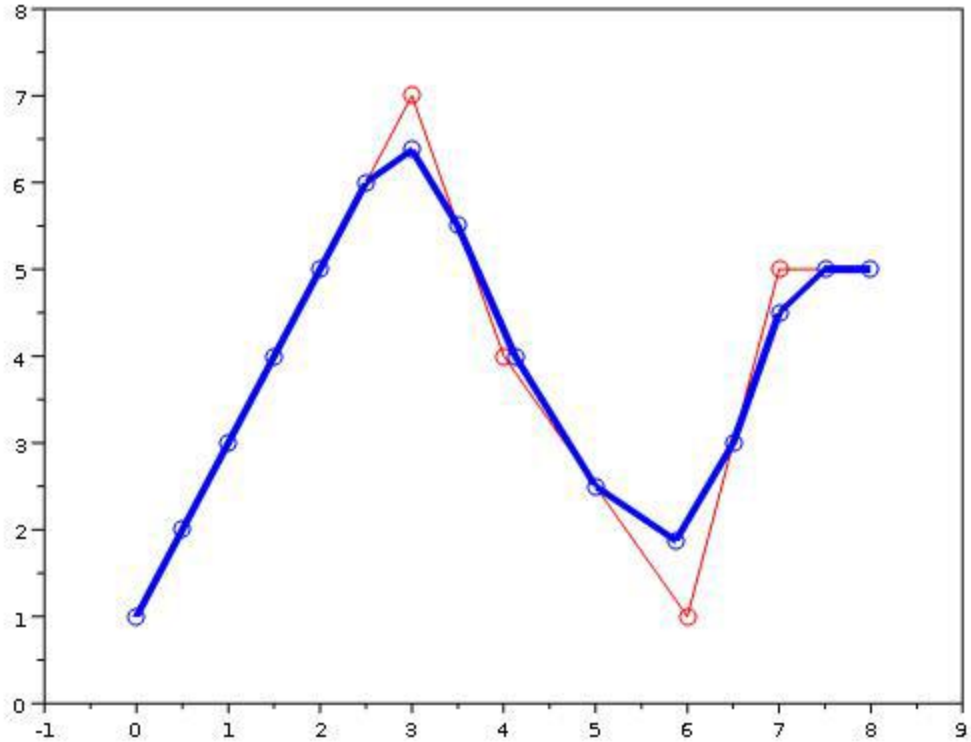


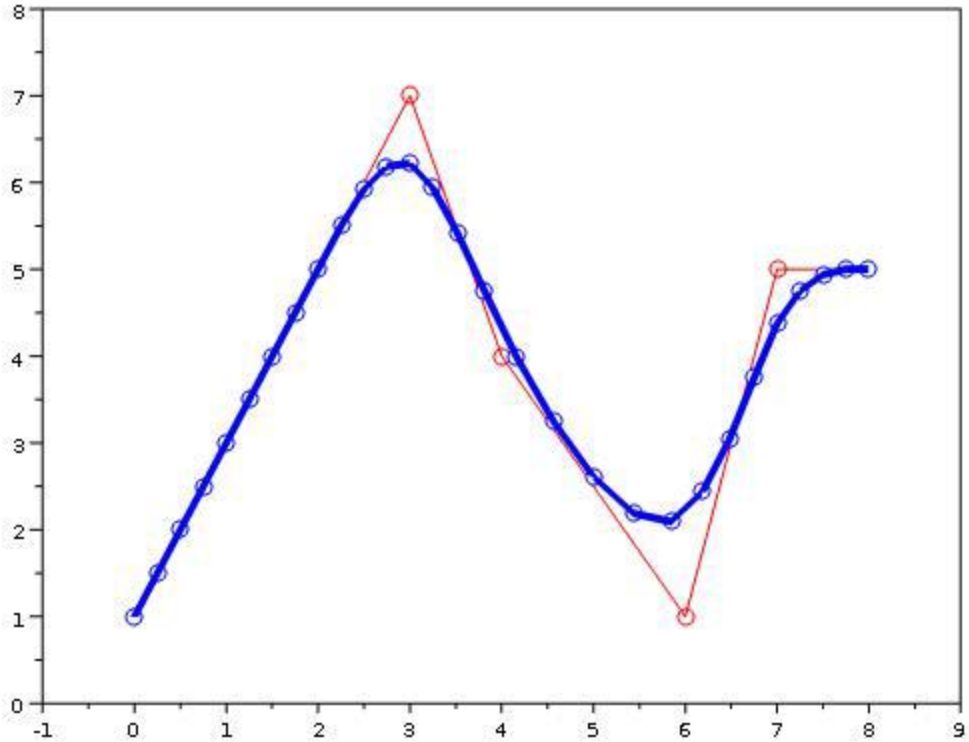
Catmull-Clark subdivision

Subdivision Curves



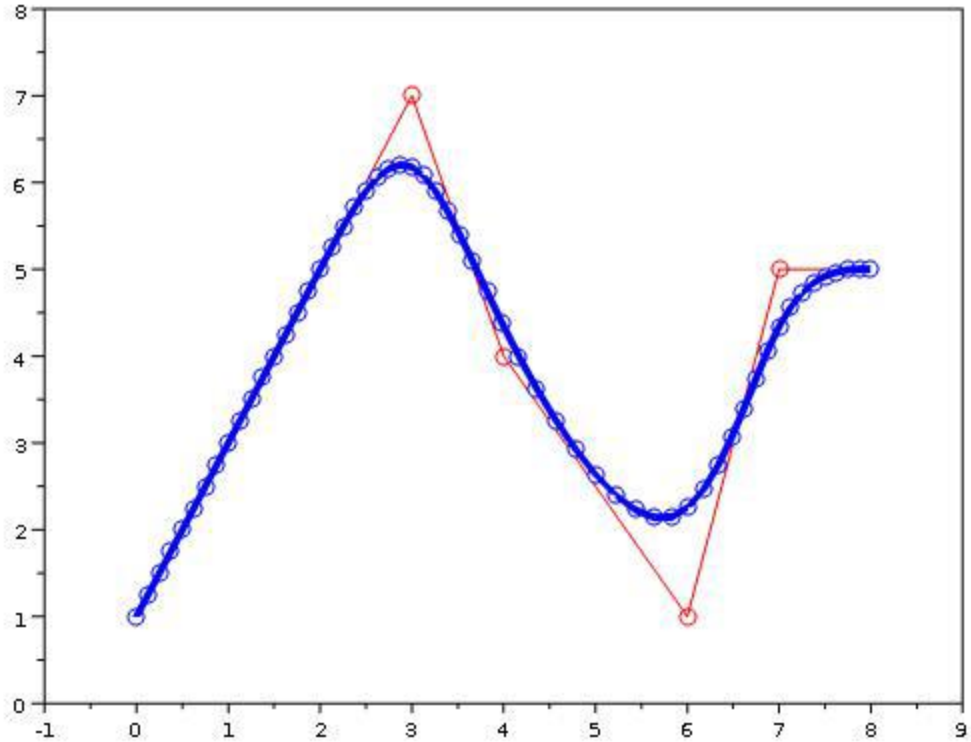
Catmull-Clark subdivision

Subdivision Curves



Catmull-Clark subdivision

Subdivision Curves



Catmull-Clark subdivision

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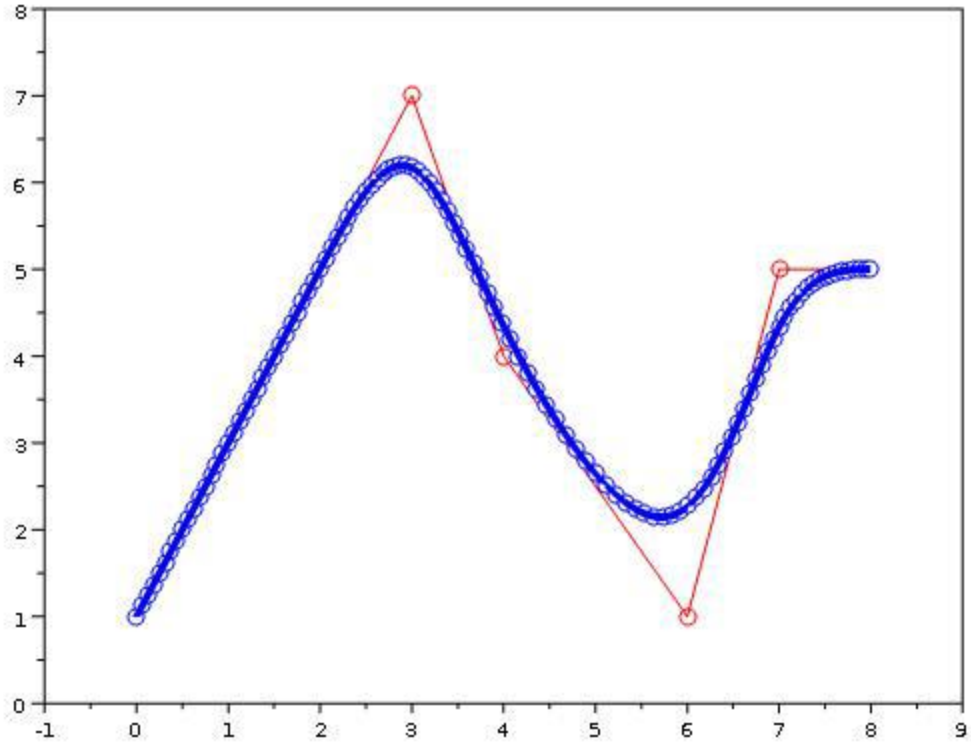

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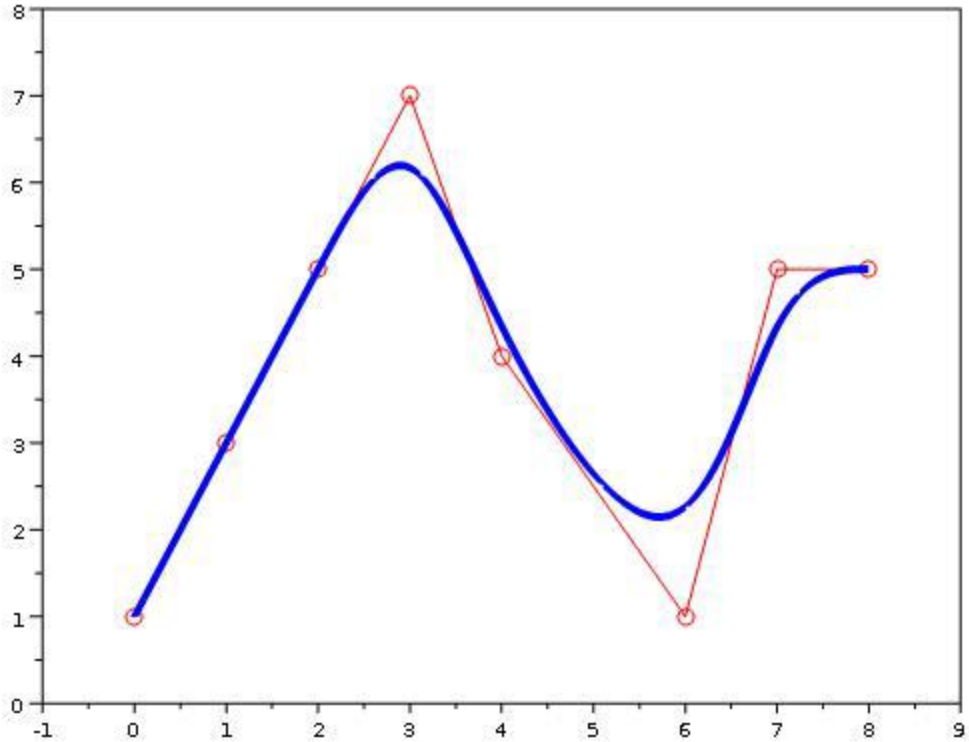

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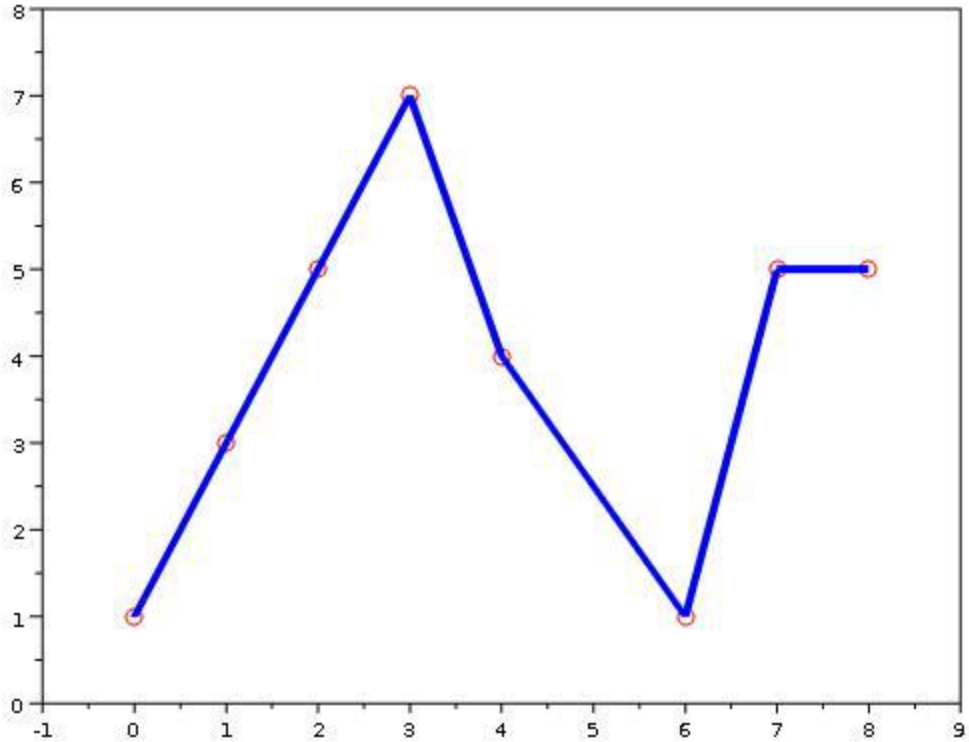
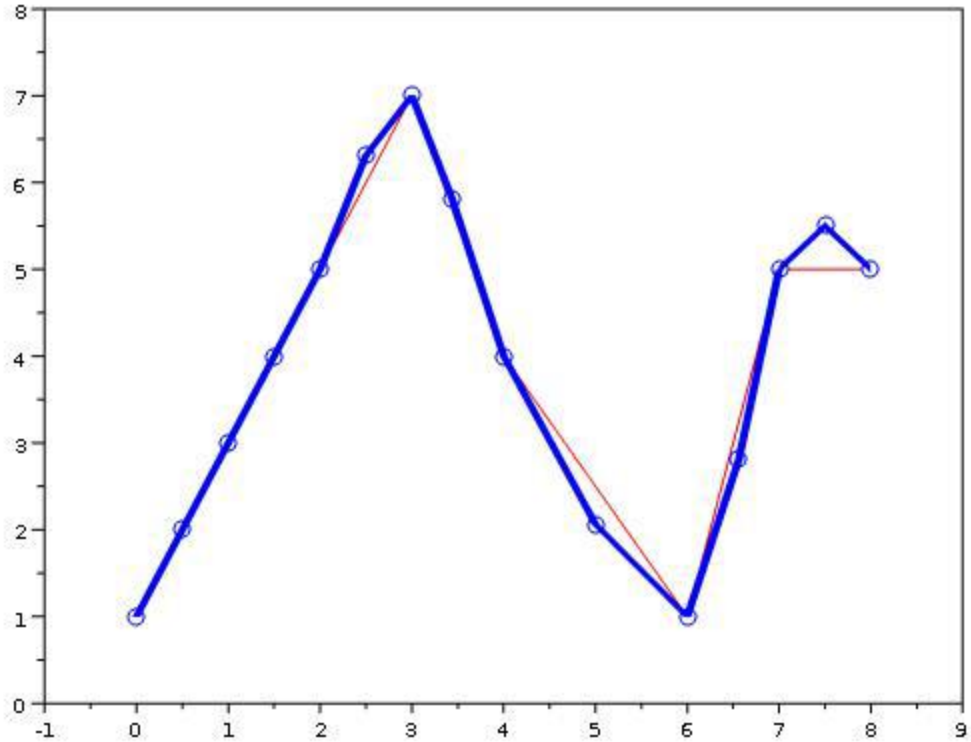


Catmull-Clark subdivision interpolation

Subdivision Curves



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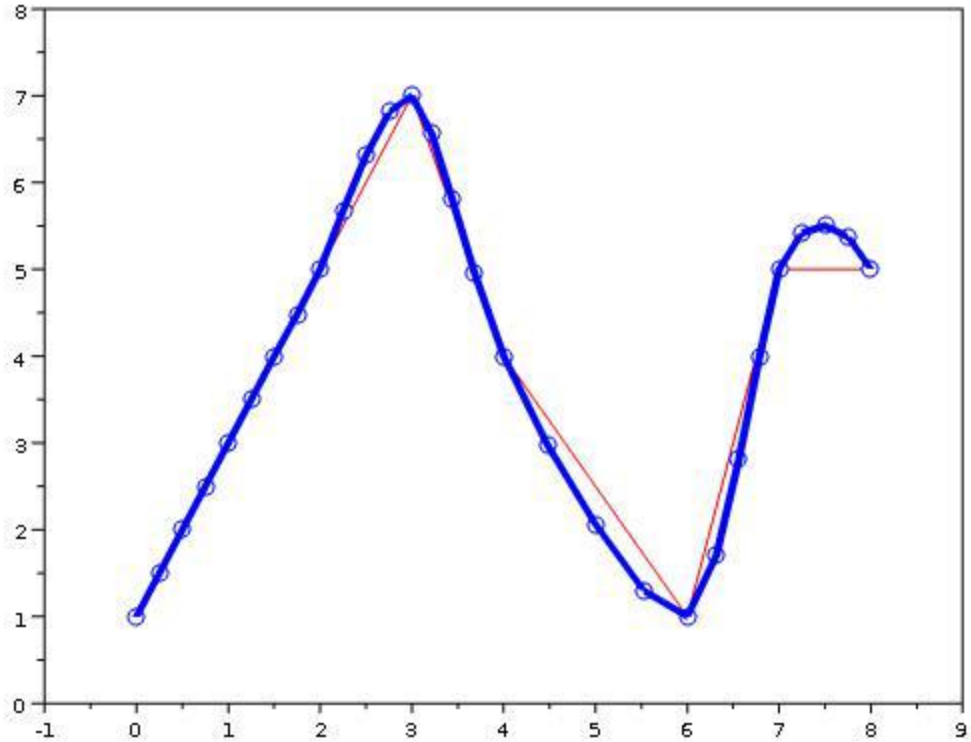

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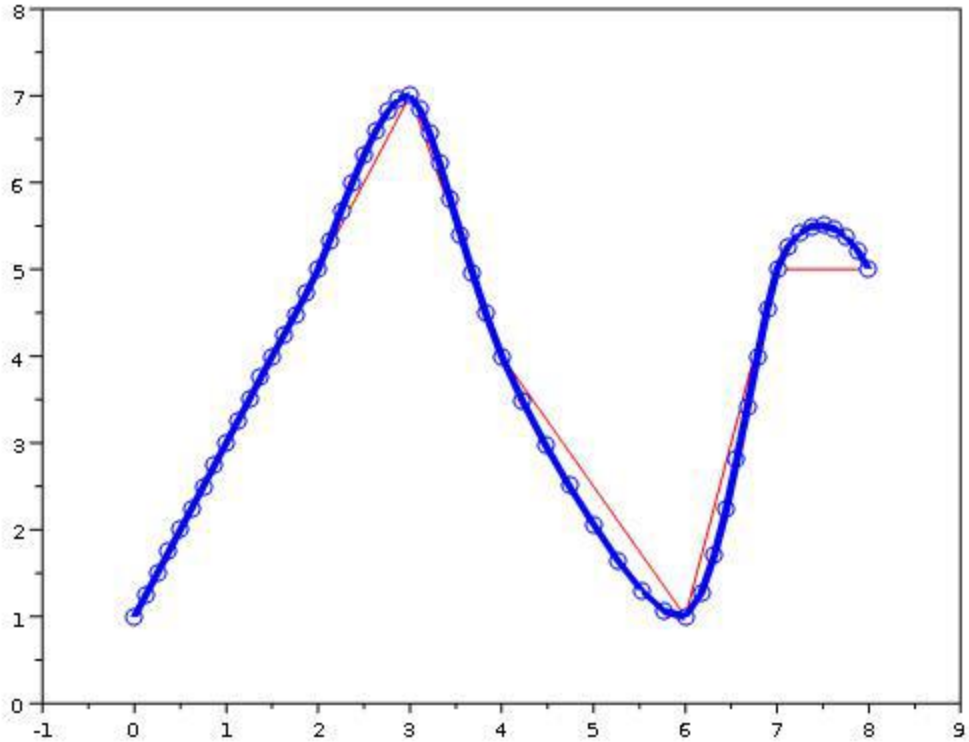
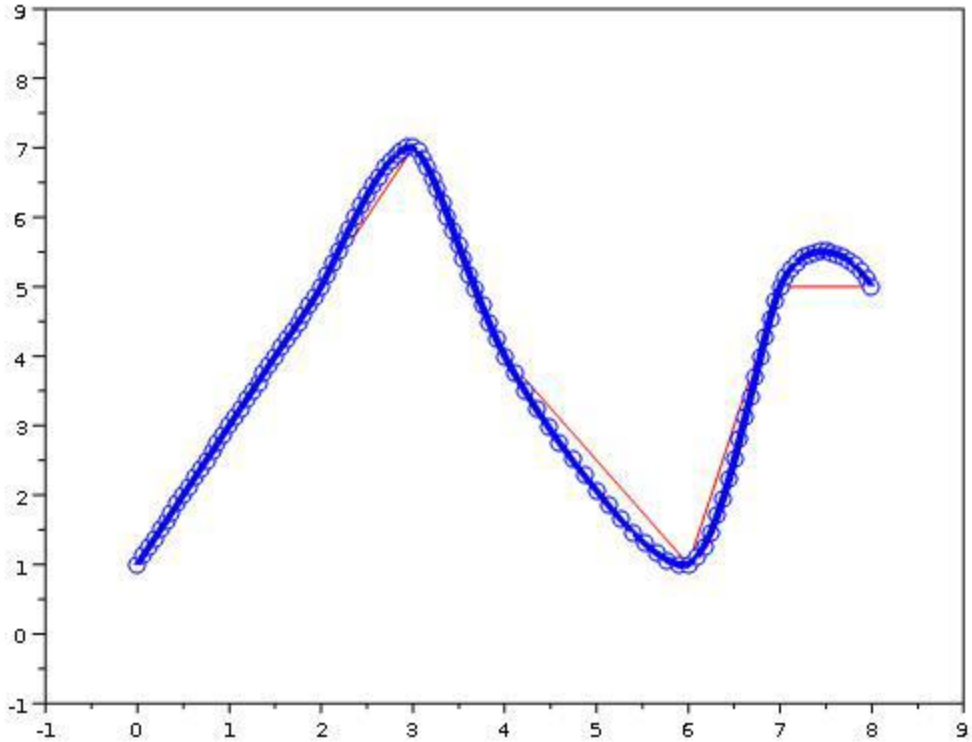


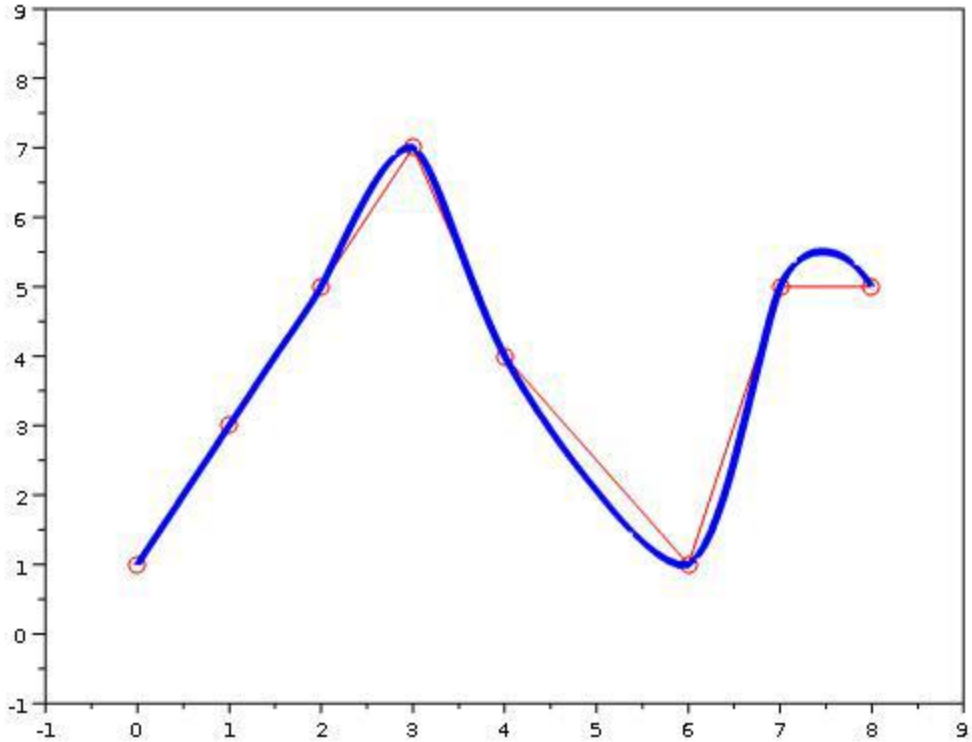
Catmull-Clark subdivision interpolation

Subdivision Curves



Catmull-Clark subdivision interpolation

Subdivision Curves



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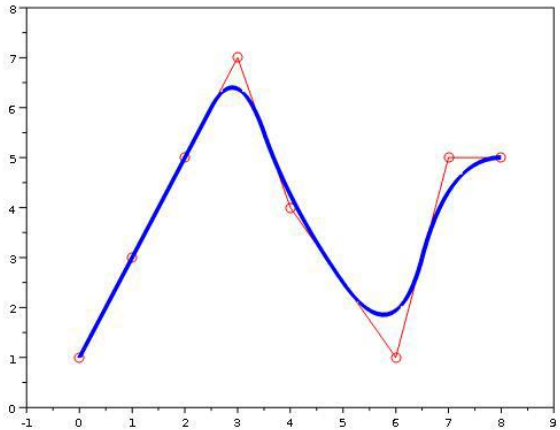
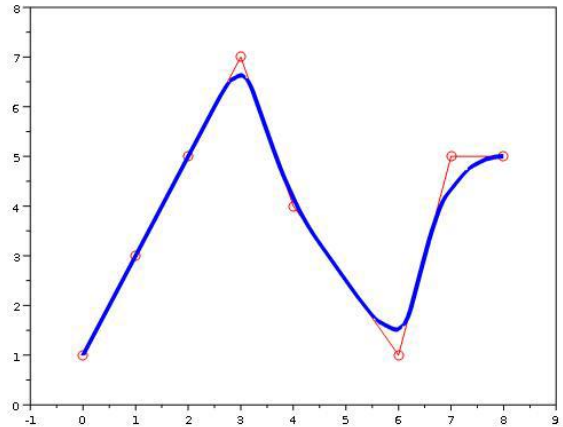
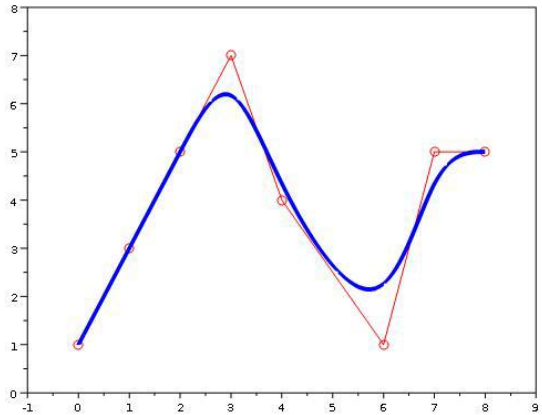


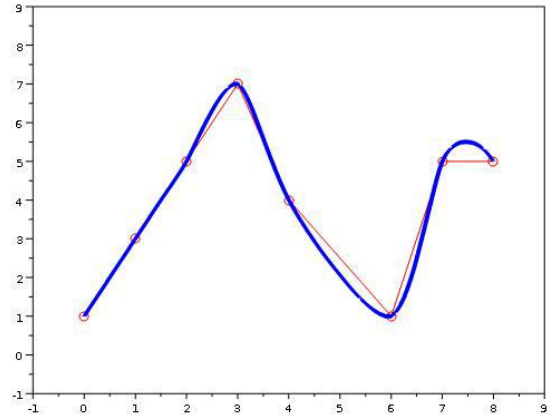
Chaikin 1



Chaikin 2

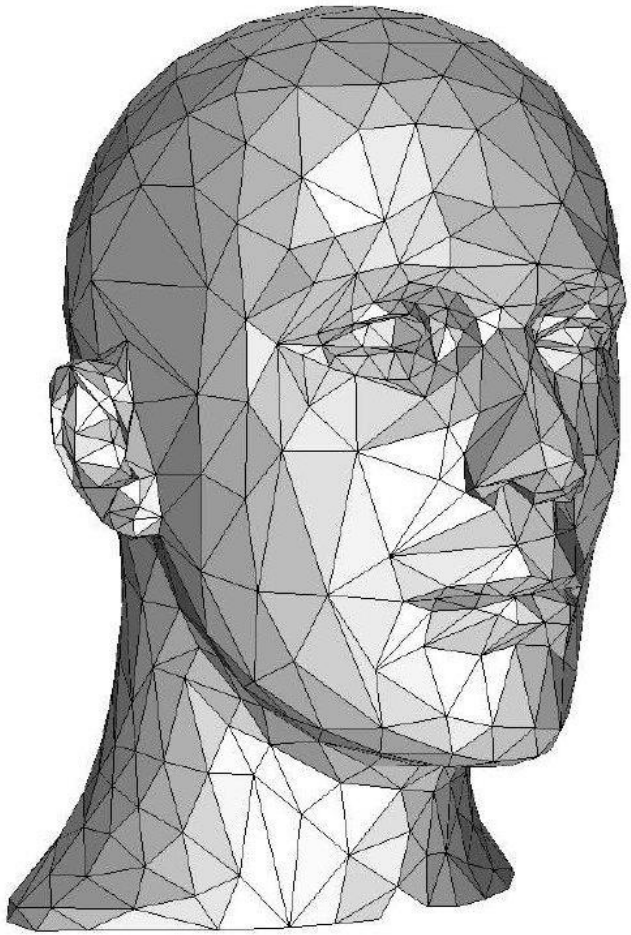


Catmull-Clark (approx)

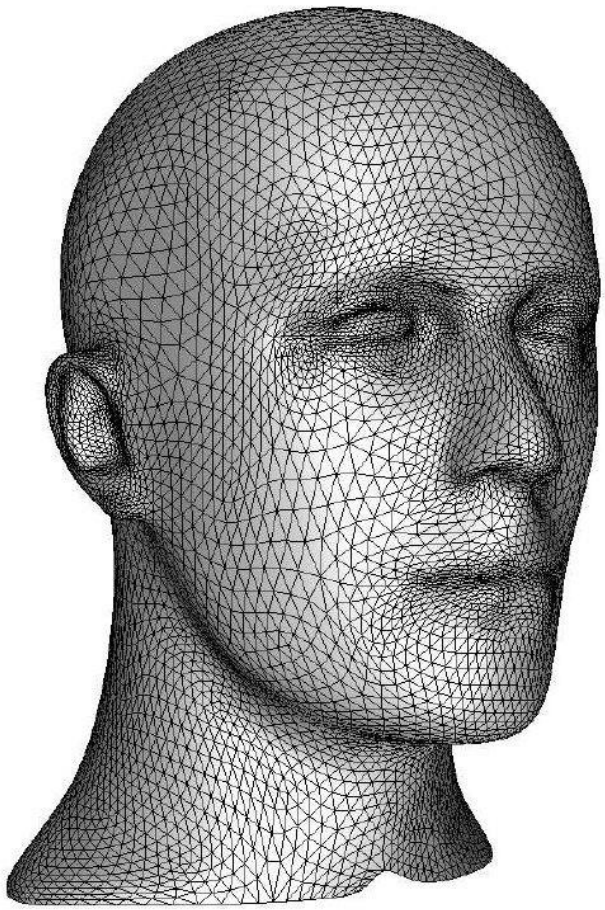


Catmull-Clark (interp)

Subdivision Surfaces

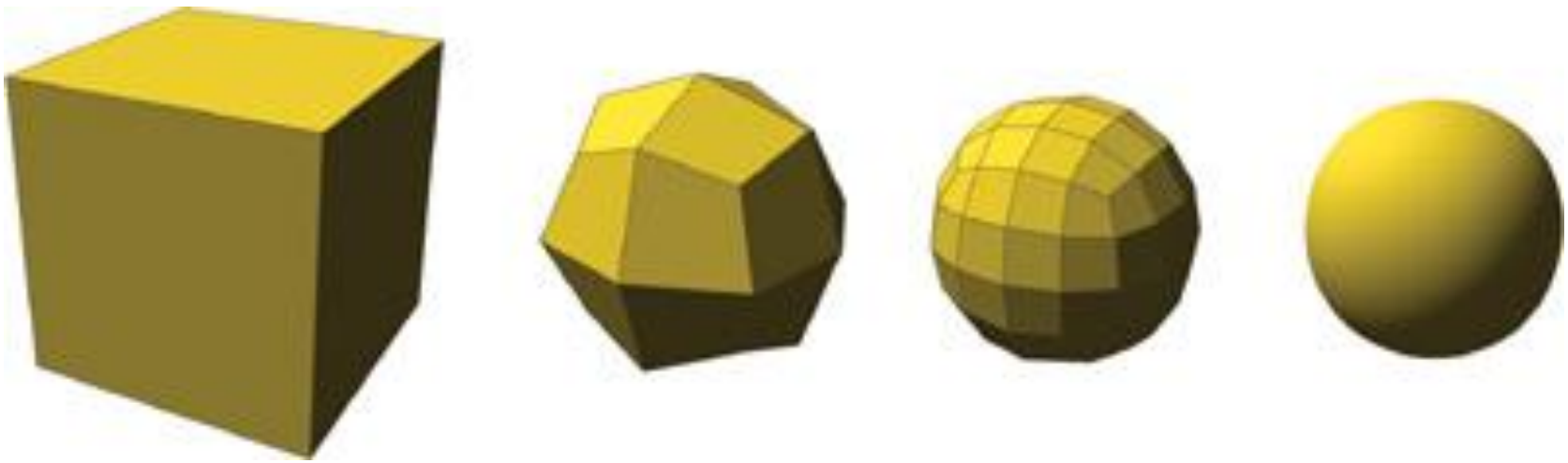


Triangle Surface



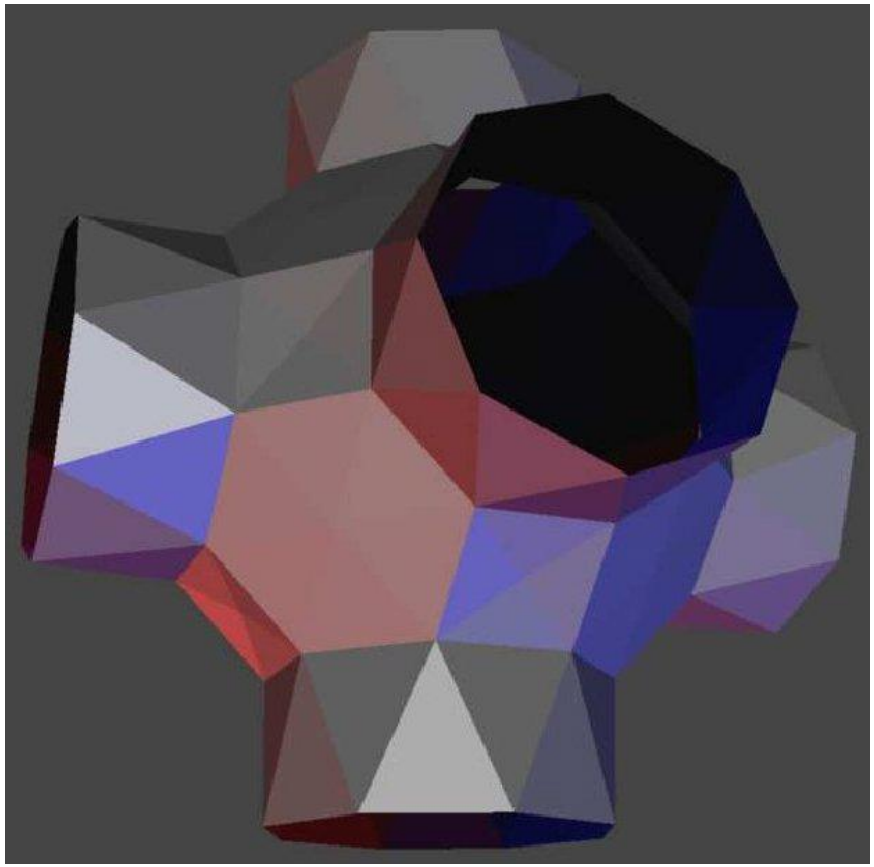
Subdivision Loop

Subdivision Surfaces

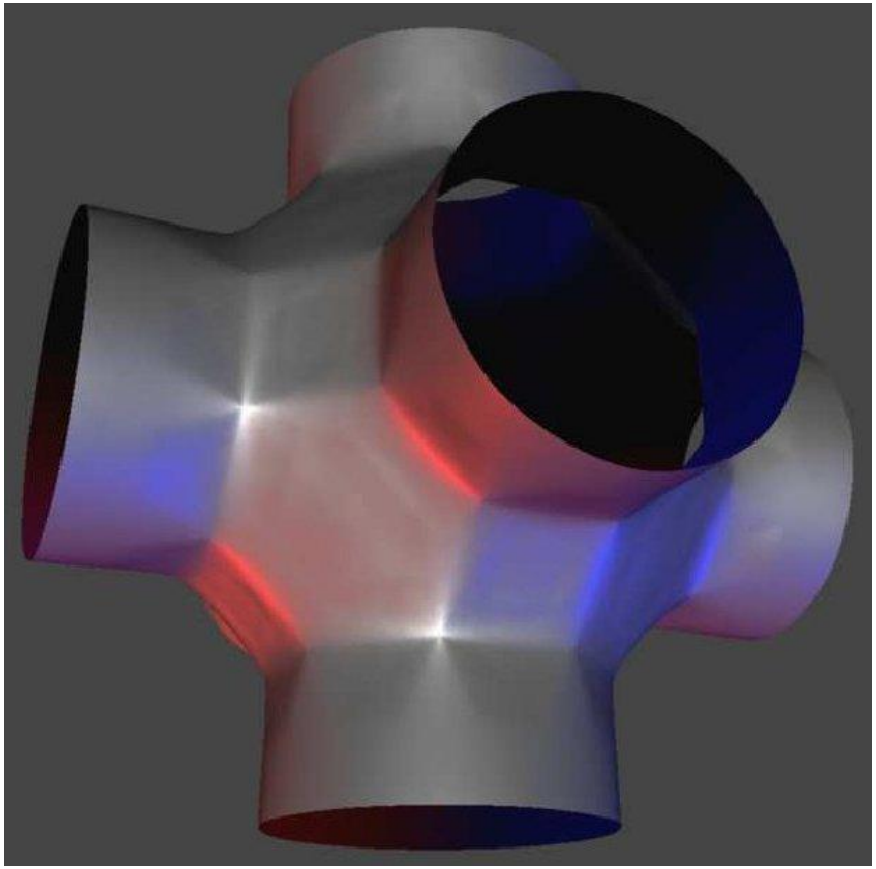


Catmull-Clark Subdivision

Subdivision Surfaces

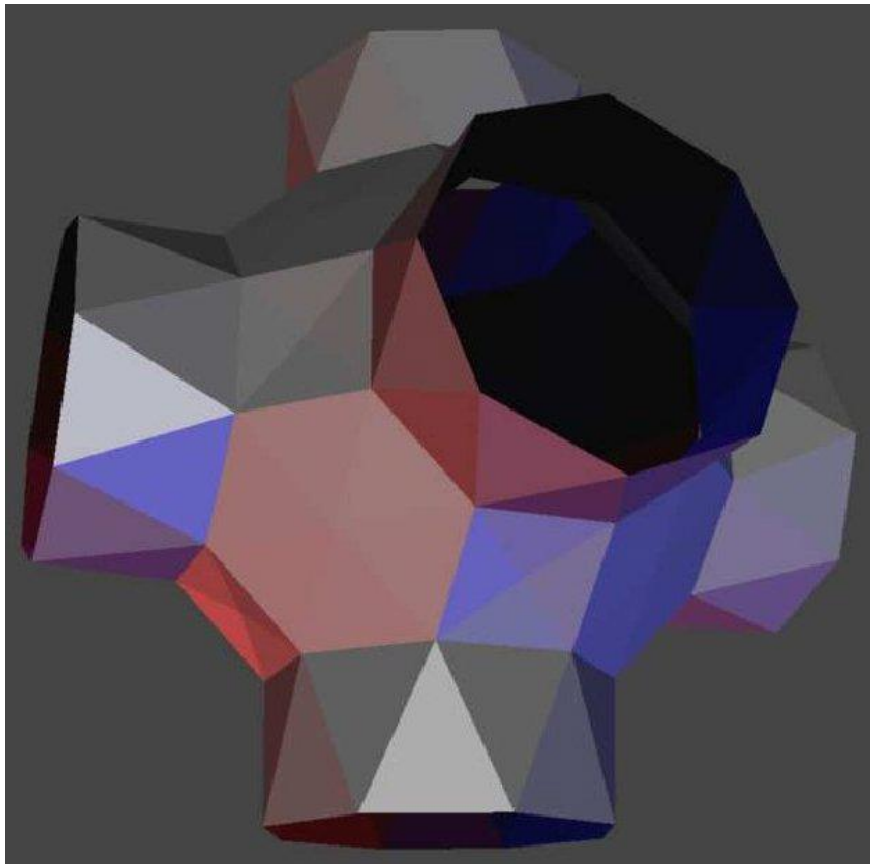


Triangulated Surface

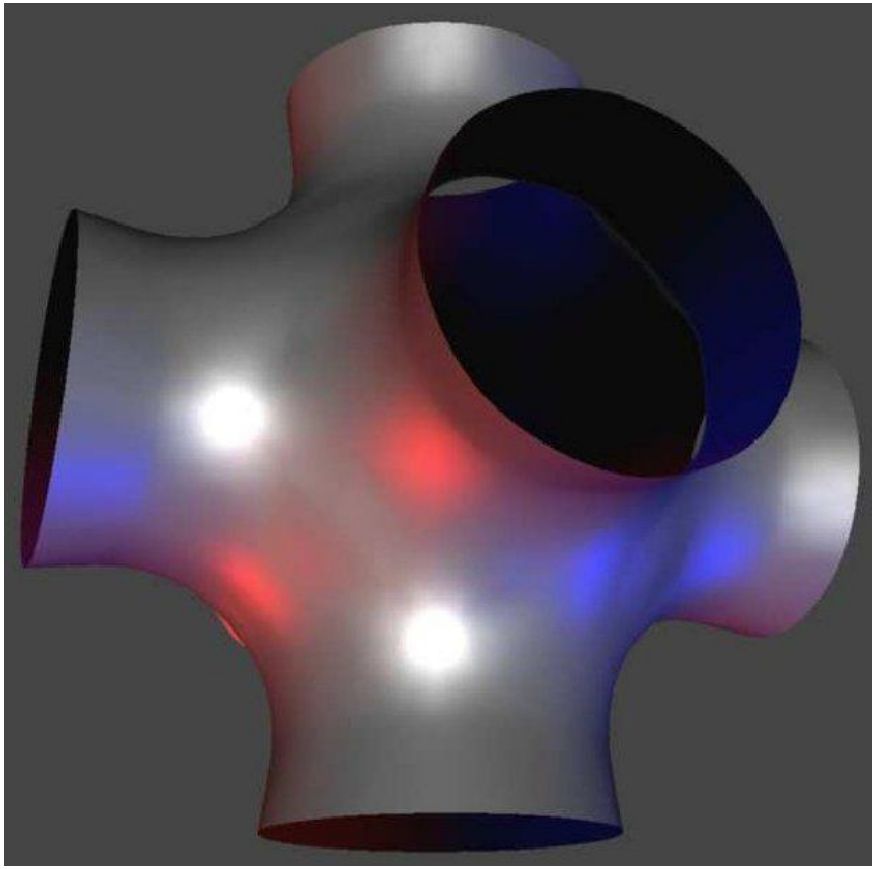


Butterfly Subdivision

Subdivision Surfaces



Triangulated Surface



Modified Butterfly Subdivision

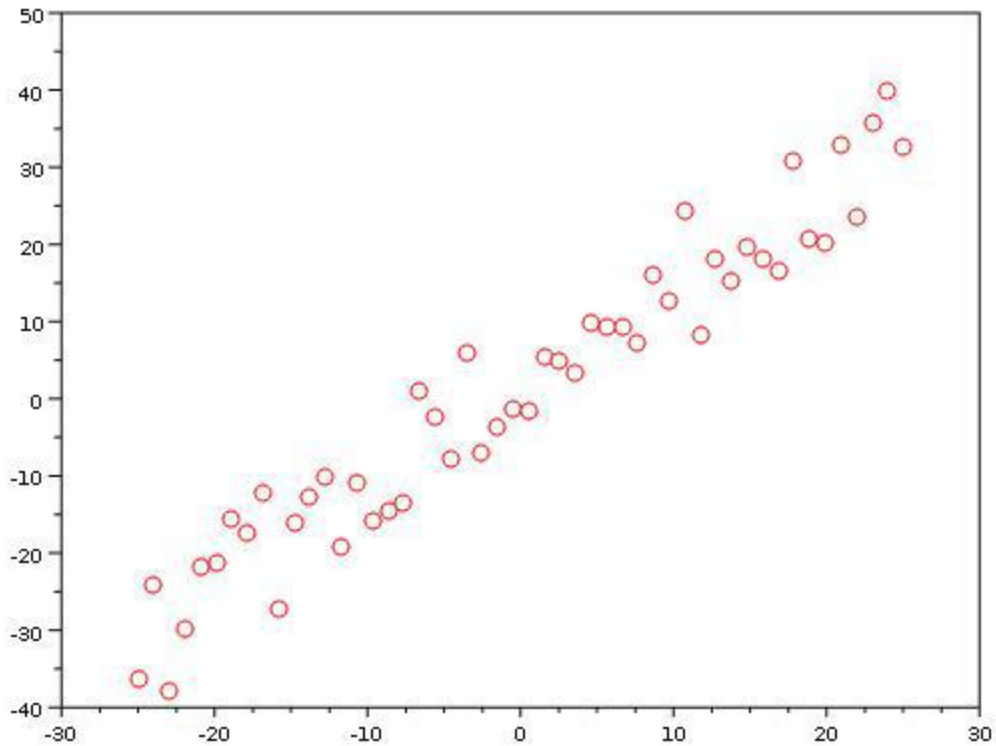
Videos

- Loop subdivision
- Catmull-Clark
- Butterfly subdivision

Comparison

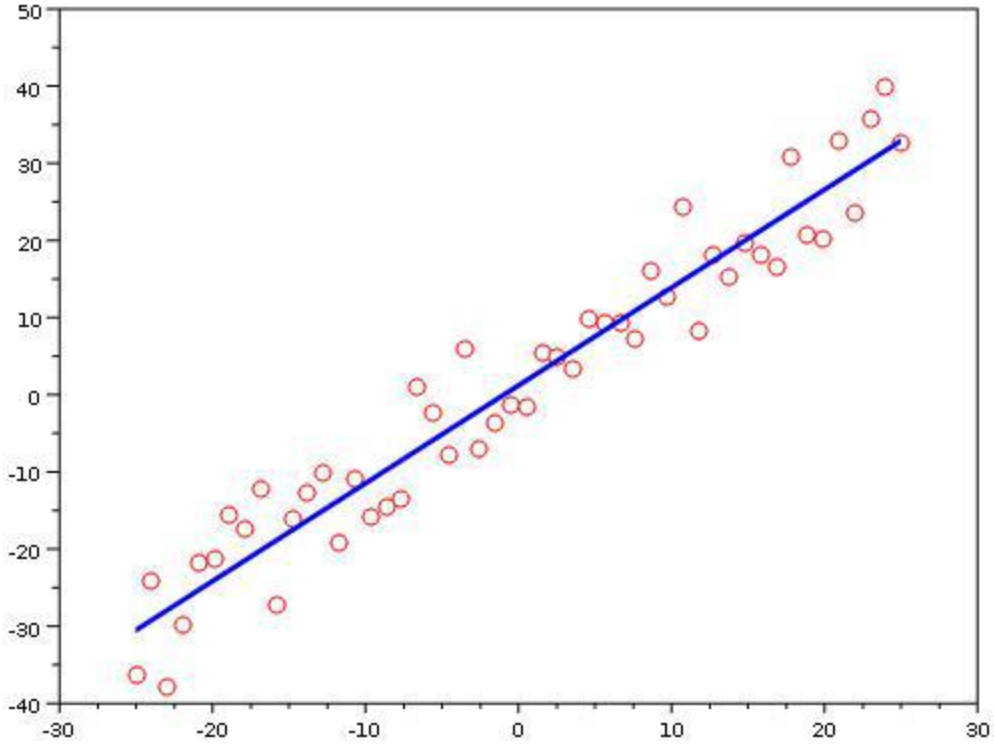
Nom	Maillage	Type	Continuité
Loop	Triangle	Approx	C^2 (C^1 s.e.)
$\sqrt{3}$	Triangle	Approx	C^2 (C^1 s.e.)
Catmull-Clark	Quadrangle	Approx	C^2 (C^1 s.e.)
Doo-Sabin	Quadrangle	Approx	C^1 (C^0 s.e.)
Butterfly	Triangle	Interp	C^1 (C^0 s.e.)
Butterfly modif.	Triangle	Interp	C^1
Kobbelt	Quadrangle	Interp	C^1

Least Squares



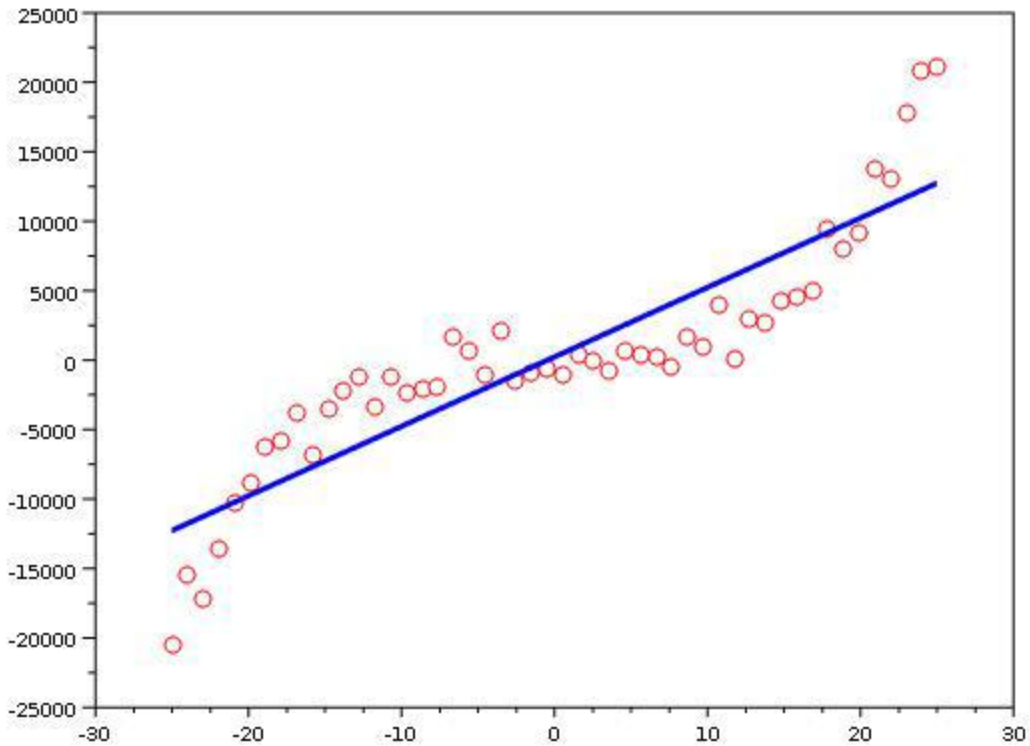
___ : $y = ax + b$

Least Squares



___ : $y = ax + b$

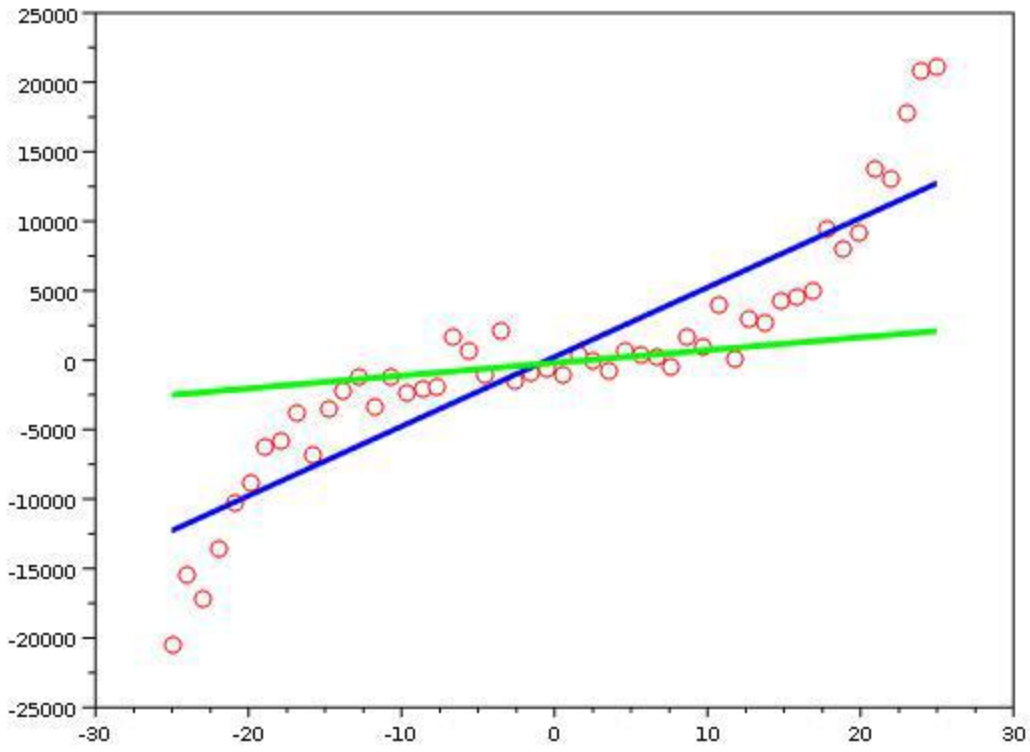
Least Squares



$y = ax + b$

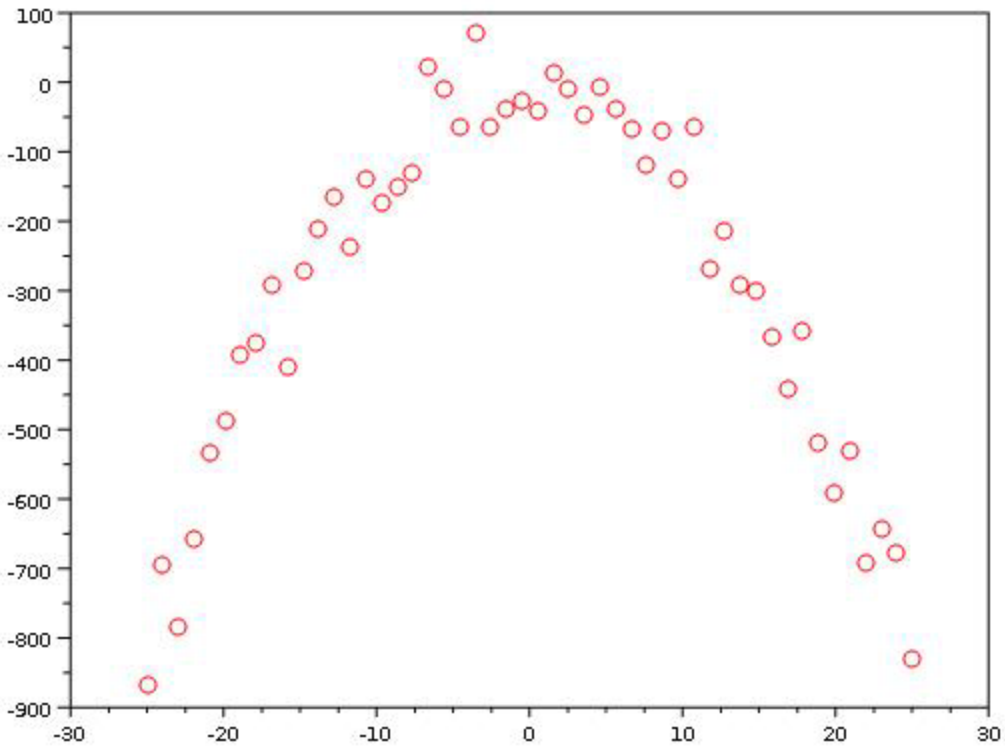
Least Squares

Gaussian



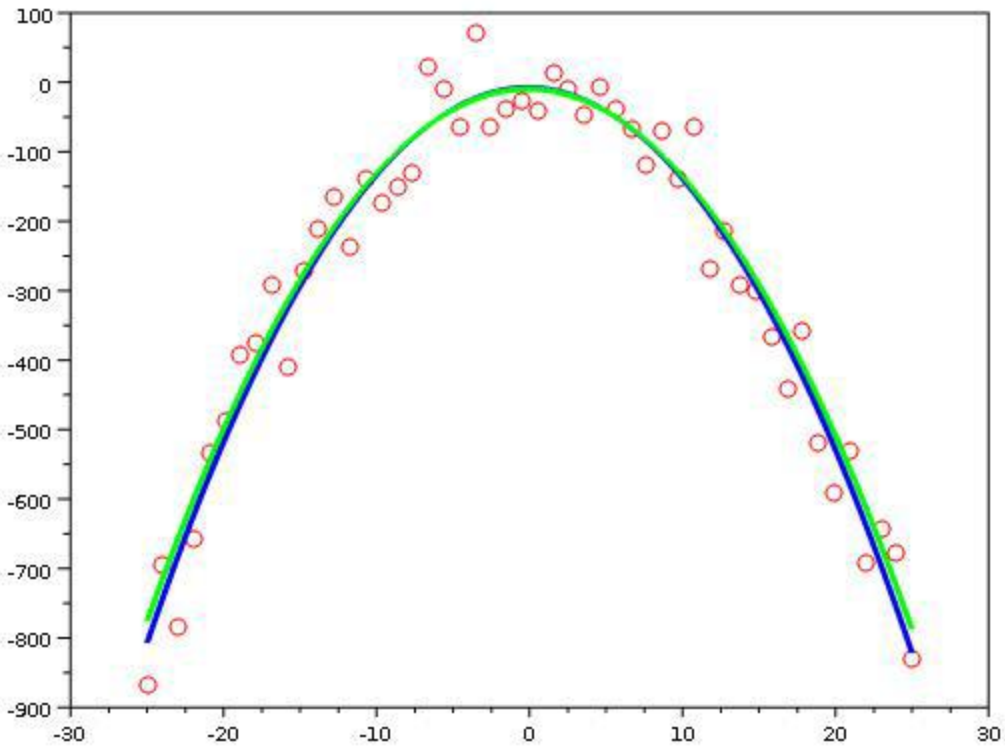
$$y = ax + b$$

Weighted Least Squares



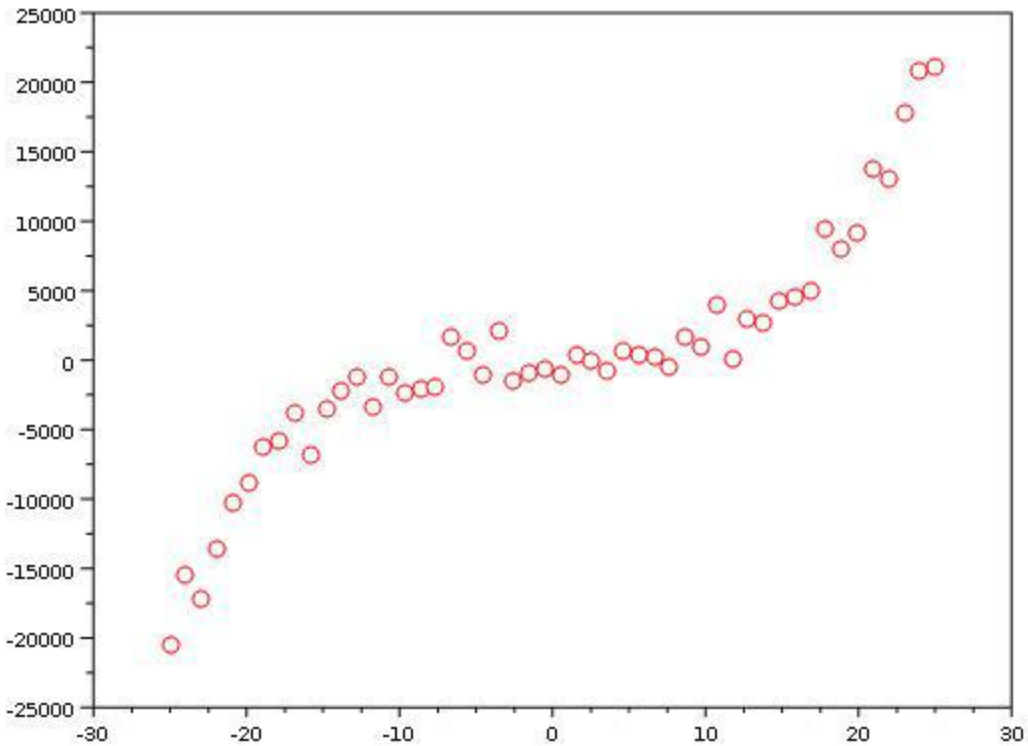
Parabola : $y = ax^2 + bx + c$

Weighted Least Squares



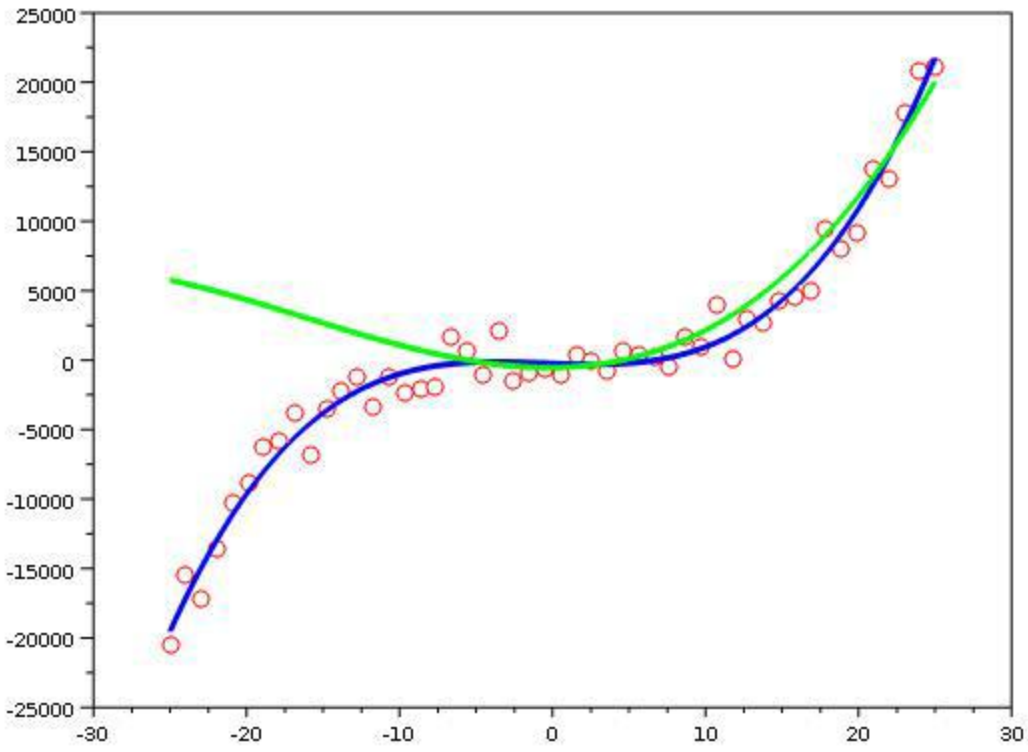
Parabola : $y = ax^2 + bx + c$

Weighted Least Squares



Cubic : $y = ax^3 + bx^2 + cx + d$

Weighted Least Squares



Cubic : $y = ax^3 + bx^2 + cx + d$

Bibliography

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- Scattered Data Interpolation and Approximation for Computer Graphics (Siggraph Asia course 2010)
- Implicit surface reconstruction from point clouds (Johan Huysmans's thesis)